Операторно-разностный метод численного решения задач МГД и его применение к моделированию магниторотационных процессов в астрофизике.

Сергей Моисеенко

ИКИ РАН, Москва

Операторно-разностная схема

Арделян Н.В., Космачевский К.В. "Математическое моделирование". М.: МГУ, 1993, с.25-44

Лагранжева, неявная, на треугольной сетке переменной структуры, полностью консервативная

Метод опорных операторов (Самарский) – сеточные аналоги основных дифференциальных операторов:

GRAD(скаляр) (дифференциальный) ~ GRAD(скаляр) (сеточный аналог)
DIV(вектор) (дифференциальный) ~ DIV(вектор) (сеточный аналог)
CURL(вектор) (дифференциальный) ~ CURL(вектор) (сеточный аналог)
GRAD(вектор) (дифференциальный) ~ GRAD(вектор) (сеточный аналог)
DIV(тензор) (дифференциальный) ~ DIV(тензор) (сеточный аналог)

НЕЯВНАЯ схема => ограничения на шаг по времени слабее, чем в явной схеме (нет условия Куранта).

Схема Лагранжева=> автоматическое сохранение углового момента.

Общие свойства скалярных, векторных и тензорных функций

$$\int_G p\nabla\cdot\vec{v}\,dV + \int_G \vec{v}\cdot\nabla p\,dV = \int_{\partial G} p\vec{v}\cdot d\vec{S},$$

$$\int_G \vec{E}\cdot(\nabla\times\vec{H})\,dV - \int_G \vec{H}\cdot\cdot(\nabla\times\vec{E})\,dV = \int_{\partial G}(\vec{H}\times\vec{E})\cdot d\vec{S},$$

$$\int_G \vec{v}\cdot\nabla A\,dV + \int_G A\cdot\nabla\vec{v}\,dV = \int_{\partial G}(A\cdot\vec{v})\cdot d\vec{S}.$$

Сеточные операторы задаются по аналогии с формулой:

$$(\nabla * A)(\vec{x}) = \frac{1}{V}\int_G(\nabla * A)\,dV = \frac{1}{V}\int_{\gamma(G)} d\vec{S} * A$$

Некоторые определения

Define linear operators $\nabla_\times{}^0$, $\nabla_\triangle\cdot$ and Φ_γ in the following form:

$$\left(\nabla_\triangle \cdot \mathbf{v}\right)_i = \frac{1}{3V_i}\sum_{l=1}^{3} N_{l,l+1}(\mathbf{v}_l R_{l,l+1} + \mathbf{v}_{l+1} R_{l+1,l}), \quad (2)$$

$$\mathbf{v} \in B_{\times,1},\ \triangle_i \in \omega_\triangle$$

$$\left(\nabla_\times{}^0 p\right)_j = \frac{1}{3V_j}\sum_{k=1}^{K_j} N_k{}' p_k, \quad p \in B^0_{\triangle,0},\ \overline{x}_j \in \omega_\times \quad (3)$$

$$(\Phi_j p)_q = \frac{1}{3V_q} N'_q p_q, \quad p \in B_{\gamma,0},\ \overline{x}_q \in \omega_\gamma, \quad (3')$$

where $R_{j_1,j_2} = r_{j_1} + r_{j_2}/2$,

$N_{j_1,j_2} = \begin{pmatrix} z_{j_2} - z_{j_1} \\ r_{j_1} - r_{j_2} \end{pmatrix}$ is a normal vector to the segment

$$[x_{j_1}, x_{j_2}],$$

$$N_k{}' = -(N_{j,k} R_{j,k} + N_{j,k+1} R_{j,k+1}),$$

$$N_q{}' = (N_{q,q+1} R_{q,q+1} + N_{q-1,q} R_{q,q-1}).$$

Introduce also the grid operator $\nabla_\times$ in the following form:

$$\forall \tilde{p} = p + p_\gamma \in B_{\triangle,0}, \quad p \in B^0_{\triangle,0}, \quad p_\gamma \in B_{\gamma,0}:$$

$$\nabla_\times \tilde{p} = \begin{cases} \nabla_\times{}^0 p + \Phi_\gamma p_\gamma, & \overline{x}_j \in \omega_\times, \\ \nabla_\times{}^0 p, & \overline{x}_j \in \omega_\times \backslash \omega_\gamma. \end{cases}$$

The operator $\nabla_\triangle \cdot$ is a grid analog for the differential operator div, $\nabla_\times$ is a grid analog for grad. The following relation is valid for the introduced operators:

$$(\nabla_\times p, \mathbf{v})_\times + (\nabla_\triangle \cdot \mathbf{v}, p)_\triangle = (\Phi_\gamma p_\gamma, \mathbf{u}_\gamma)_\gamma$$

where $p \in B^0_{\triangle,0}$, $\mathbf{v} \in B_{\times,1}$, $p_\gamma \in B_{\gamma,0}$, $\mathbf{v}_\gamma \in B_{\gamma,1}$.

Операторы $\nabla_\times$ и $-\nabla_\Delta\cdot$ сопряжены.

оператор $\nabla \cdot \nabla$ самосопряженный

Операторно-разностная схема для уравнений гидродинамики

$$\rho_i V_i = {\rho_i}^0 {V_i}^0 = m_i, \quad \forall \Delta_i \in \omega_\Delta$$

$$\mathbf{x}_{jt} = {\mathbf{u}_j}^{(0.5)}, \quad \forall x_j \in \omega_\times$$

$$\rho_j \mathbf{u}_{jt} = -(\nabla_\times g)_j - q\rho_j \big(S_\times(\nabla_\Delta \Phi)\big)_j, \quad \forall x_j \in \omega_\times$$

$$\rho_i \varepsilon_{it} = -g_i \big(\nabla_\Delta \cdot \mathbf{u}^{(0.5)}\big)_i, \quad \forall \Delta_i \in \omega_\Delta$$

$$g = p^{(\alpha)} + \omega, \quad \omega = -\frac{\nu}{\eta}\eta_t, \quad \forall \Delta_i \in \omega_\Delta$$

$$\eta_i = 1/\rho_i = T_i/p_i, \quad \varepsilon_i = T_i/(\gamma - 1), \quad \forall \Delta_i \in \omega_\Delta$$

$$(\nabla_\times \cdot \nabla_\Delta \Phi)_j = \rho_j,$$

$$\rho_j = (S_\times \rho)_j = \frac{1}{3V_j^\times} \sum^{K_j} \rho_k V_k^\Delta, \quad \forall x_j \in \omega_\times .$$

где $\mathbf{u}_{jt} = \left\{ u_{jt}^r - \frac{u_j^{\varphi(0.5)}}{r_j^{(0.5)}} u_j^{\varphi(0.5)}, \quad u_{jt}^\varphi + \frac{u_j^{\varphi(0.5)}}{r_j^{(0.5)}} u_j^{r(0.5)}, \quad u_{jt}^z \right\}$

Операторно-раностная схема для уравнений МГД

$$\vec{x}_t = \vec{v}^{(\sigma_1)}, \qquad \left(\frac{1}{\varrho}\right)_t = \nabla^m_\Delta \cdot \vec{v}^{(\sigma_1)},$$

$$\vec{v}_t + A_v = -\nabla^m_x\left(\tilde{g} + \frac{\tilde{\vec{H}}^{(1)} \cdot \tilde{\vec{H}}}{2}\right) + \tilde{\nabla}^m_x \cdot \left(\tilde{\vec{H}}^{(\sigma_2)} \tilde{\vec{H}}^{(0.5)}\right),$$

$$(A_v)_j = \frac{v^{(0.5)}_{\varphi,j}}{r^{(0.5)}_j}\left(v_r^{(0.5)}\vec{e}_\varphi - v_\varphi^{(0.5)}\vec{e}_r\right)_j, \qquad \vec{x}_j \in \omega_x,$$

$$g = p^{(\sigma_0)} - \nu_\varrho \nabla^m_\Delta \cdot \vec{v}^{(\sigma_1)}, \qquad \nu_\varrho = \nu(S_x\varrho).$$

$$\left(\frac{\vec{H}}{\varrho}\right)_t + A_H = \vec{H}^{(\sigma_2)} \cdot \tilde{\nabla}^m_\Delta \vec{v}^{(0.5)} - \nabla^m_\Delta \times \vec{E},$$

$$(A_H)_i = \frac{s_i\left(S_\Delta(v_\varphi^{(0.5)})\right)_i}{m_i}\left(H_r^{(0.5)}\vec{e}_\varphi - H_\varphi^{(\sigma_2)}\vec{e}_r\right)_i, \qquad \Delta_i \in \omega_\Delta,$$

$$\vec{E} = \frac{1}{\sigma_\Delta}\nabla^m_x \times \tilde{\vec{H}}^{(\sigma_3)}, \qquad \sigma_\Delta = (S_x \varrho)^{-1}\sigma$$

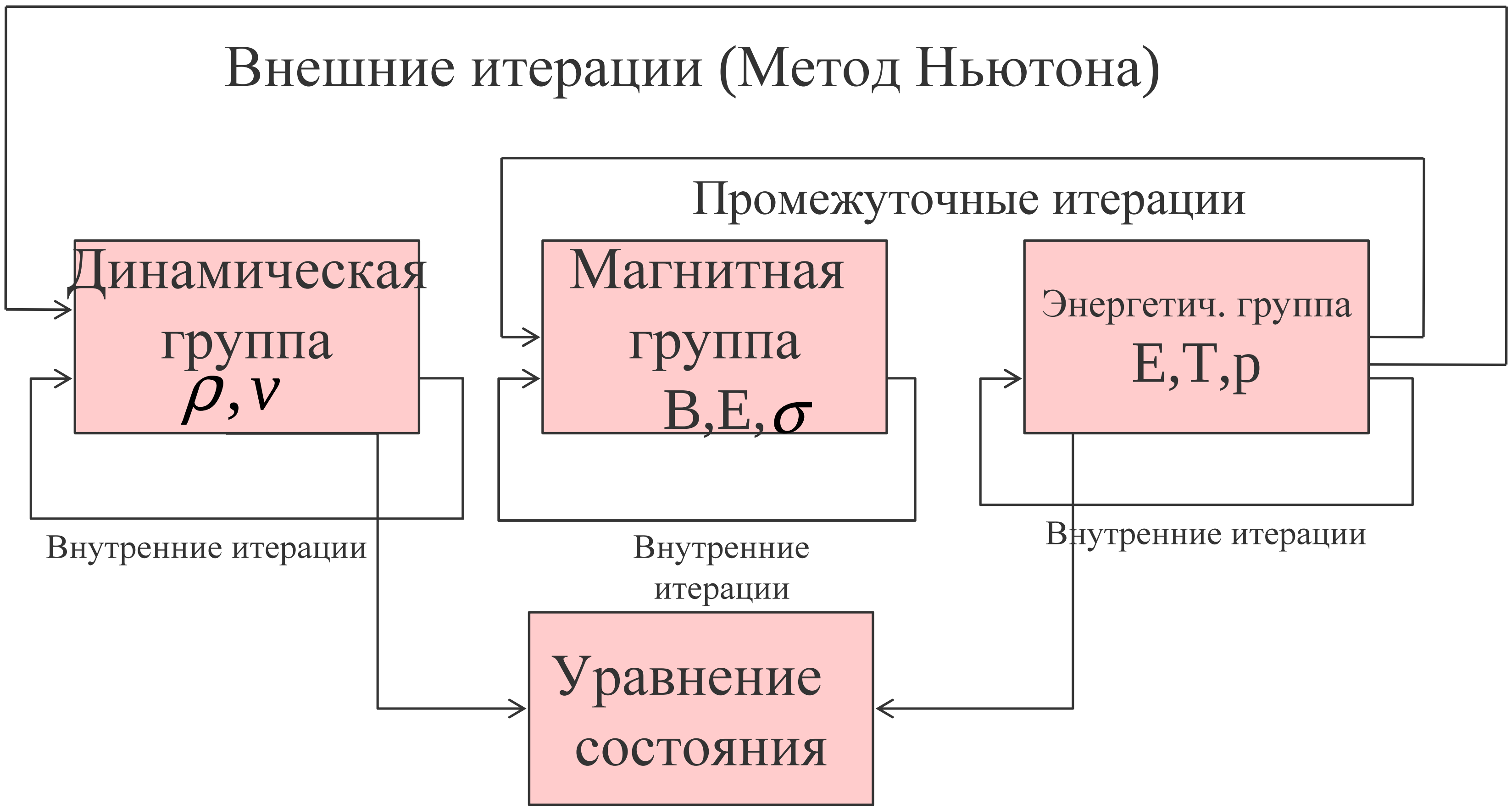
$$\varepsilon_t = -p\nabla^m_\Delta \cdot \vec{v}^{(\sigma_2)} - \nabla^m_\Delta \cdot \vec{W} + S_\Delta\left(\vec{E} \cdot \nabla^m_x \times \tilde{\vec{H}}^{(0.5)}\right),$$

$$\vec{W} = -\kappa_\varrho \nabla^m_x T^{(\sigma_4)}, \qquad \kappa_\varrho = (S_x\varrho)\kappa$$

$$p = P(\varrho, T), \qquad \varepsilon = \mathcal{E}(\varrho, T).$$

Общая схема применения неявного МГД метода

(Самарский, Попов)

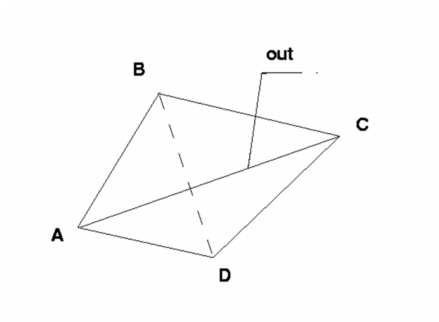


j-шаг ⟶ j+1-шаг ⟶ j+2-шаг

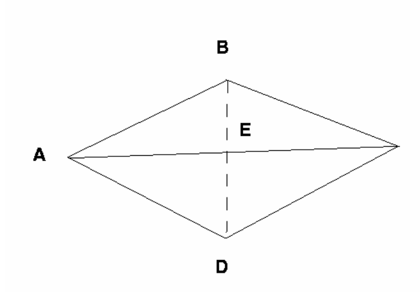
Операторы самосопряженные ⇨ Разреженные матрицы симметричны, положительно определены

Матрицы обращаются методом сопряженных градиентов(итерационный).

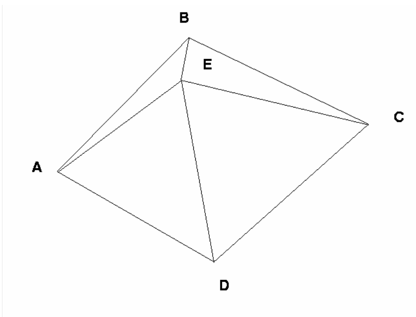
Перестройка сетки



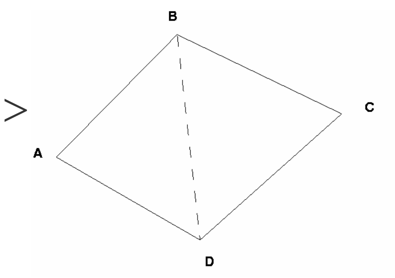
Элементарная перестройка: связь BD вводится вместо связи AC . Общее количество узлов и ячеек не меняется.



Добавление узла на середину связи:
узел E добавляется к существующим узлам ABCD
На середину связи BD , появляются 2 связи
AE и EC, общее число ячеек увеличивается на 2 ячейки.



=>



Удаление узла: узел E удаляется из сетки, общее число ячеек увеличивается на 2 ячейки.

Интерполяция сеточных функция на новую сетку (локальная):

Следует делать *консервативно*. Условная минимизация функционалов.

$\rho_{old,k}$ – плотность на старой структуре,

$\rho_{\text{int},i}$ – плотность на старой стр-ре, интерполированная на новую структуру,

$\rho_{new,i}$ – плотность на новой структуре.

Основной функционал:

$$\min_{i=\overline{1,N}} F(\rho_{new,i}) = (\rho_{new,i} - \rho_{\text{int},i})^2$$

при условии:

$$\sum_k V_{old,k}\rho_{old,k} = \sum_i V_{new,i}\rho_{new,i} = m = const$$

(условие сохранения массы)

Старая форма функционала:

$$\min_{i=\overline{1,N}} F(\rho_{new,i}) = \alpha(\rho_{new,i} - \rho_{\text{int},i})^2 + \underline{(1-\alpha)(\text{grad}(\rho_{new,i}) - \text{grad}(\rho_{\text{int},i}))^2}$$

Не работает на ударных волнах☹

Критерии адаптации сетки

Introduce function:

$$f(\rho_k,\ \text{grad}\ \rho_k) = \alpha/(\rho_k + \varepsilon) + \beta/(|\text{grad}\rho_k| + \varepsilon), \qquad (5)$$

where $0 < \varepsilon << 1$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta = 1$. In the limiting cases:

$\alpha = 1, \beta = 0$, f is a grid function of the density only;

$\alpha = 0, \beta = 1$, f is a grid function of the density gradient only.

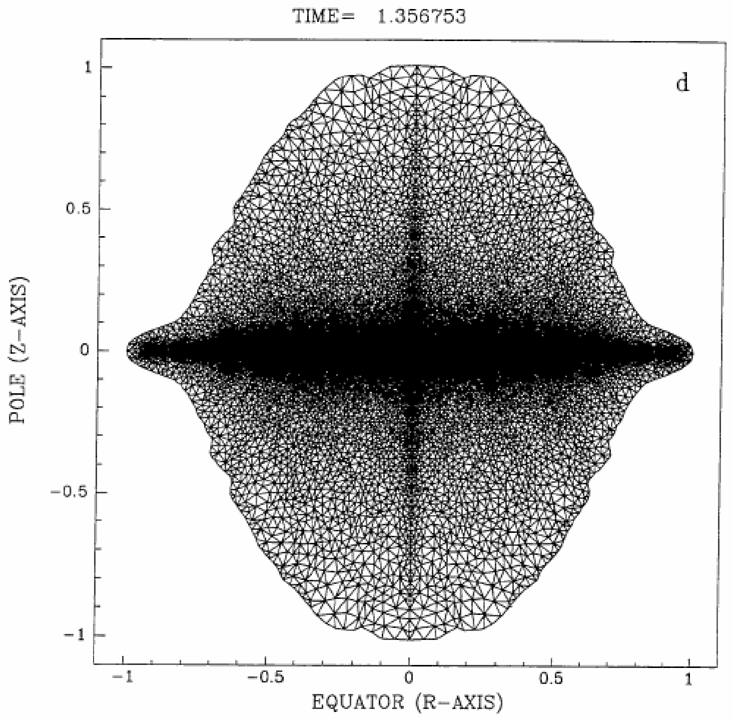
Let s_k be the area of triangle cell number k. S is the area of the whole computational domain, so that

$$S = \sum_{k=1}^{N} s_k.$$

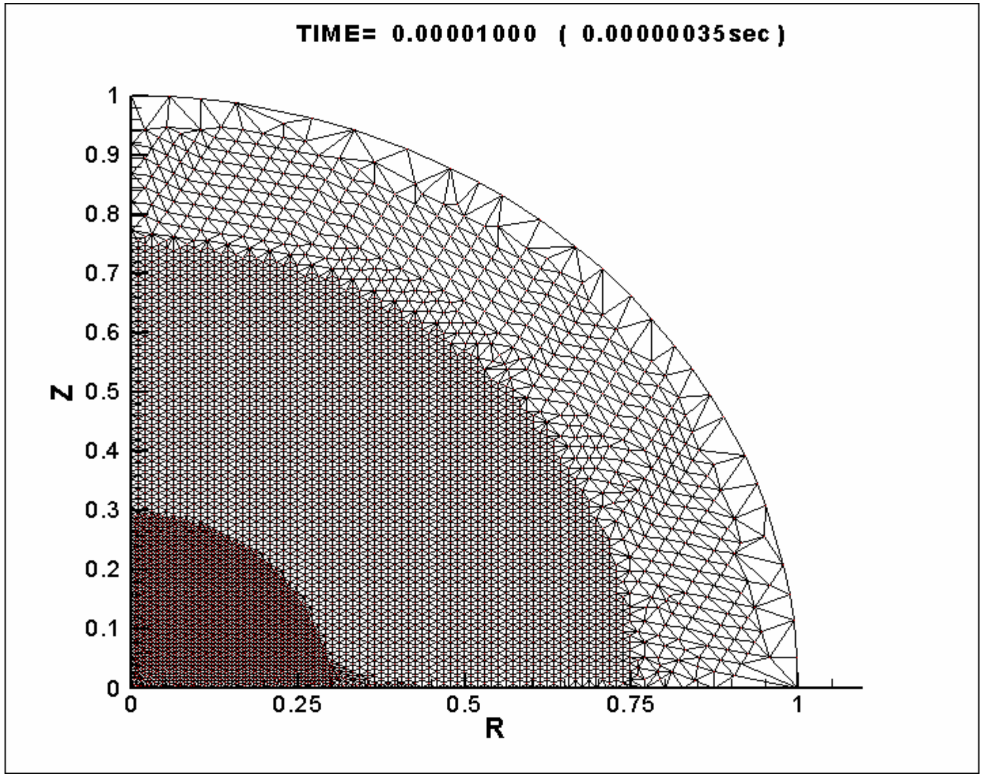
Introduce restrictions on the length of the cell side as a criterion for dynamic grid adaptation. Consider the formula for the characteristic length l_k of the side of an equilateral triangle of area s_k as a function of the density, the density gradient and (implicitly) the r, z coordinates:

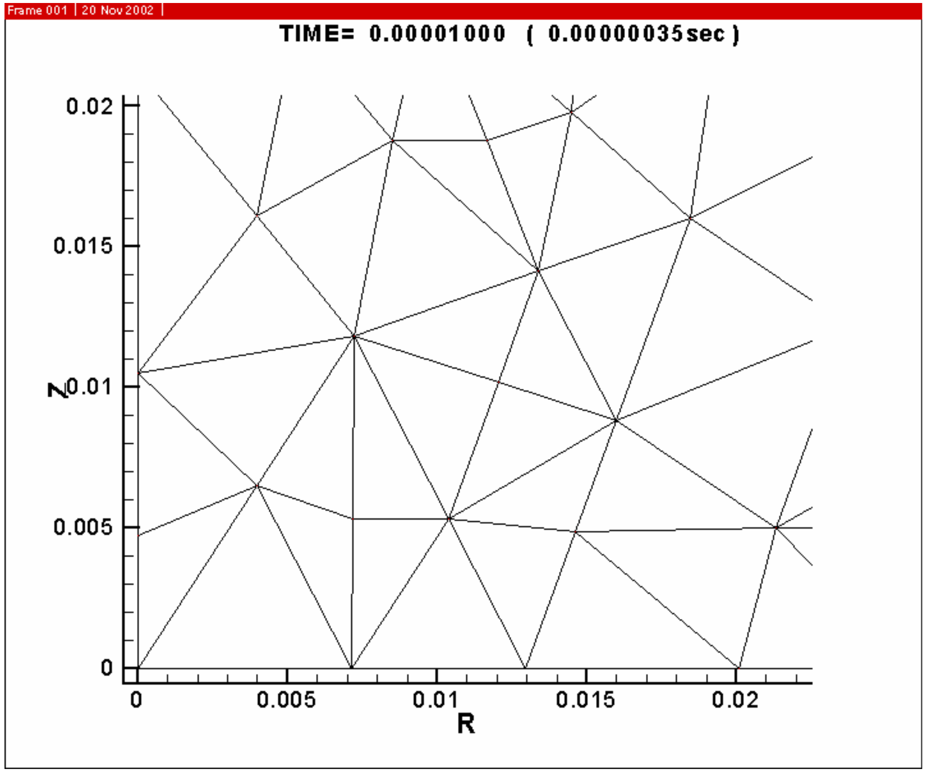
$$l_k = 2\sqrt{\frac{s_k}{3}}, \qquad s_k = \frac{f(\rho_k,\ \text{grad}\ \rho_k)}{\sum_{n=1}^{N} f(\rho_n,\ \text{grad}\ \rho_n)} S, \qquad (6)$$

Пример применения критериев:



Пример эволюции треугольной сетки

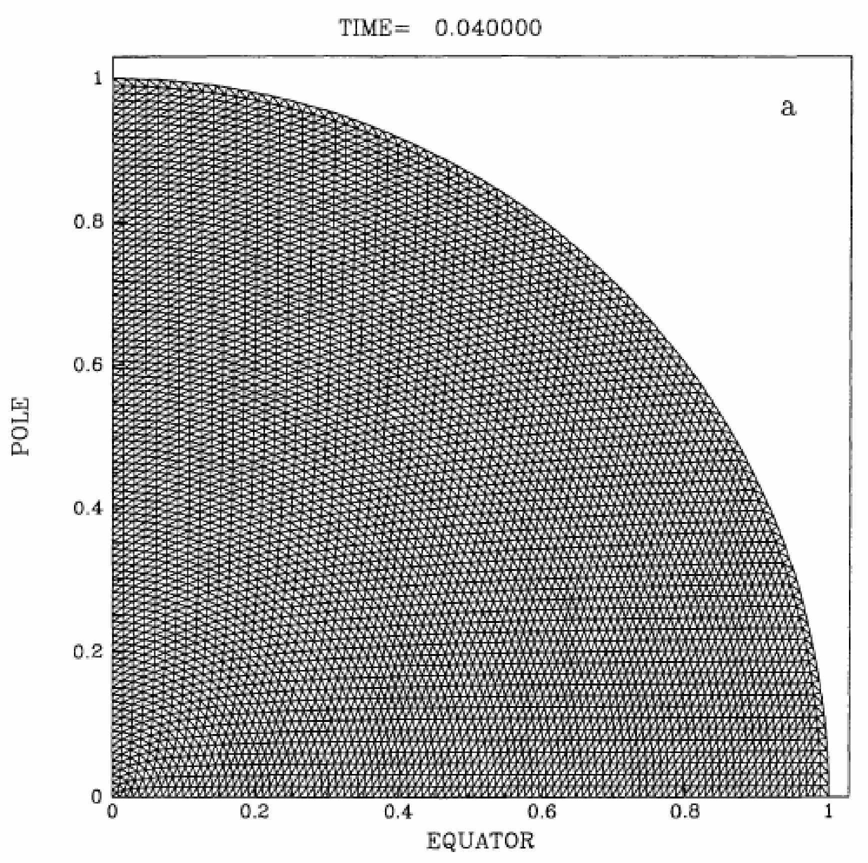


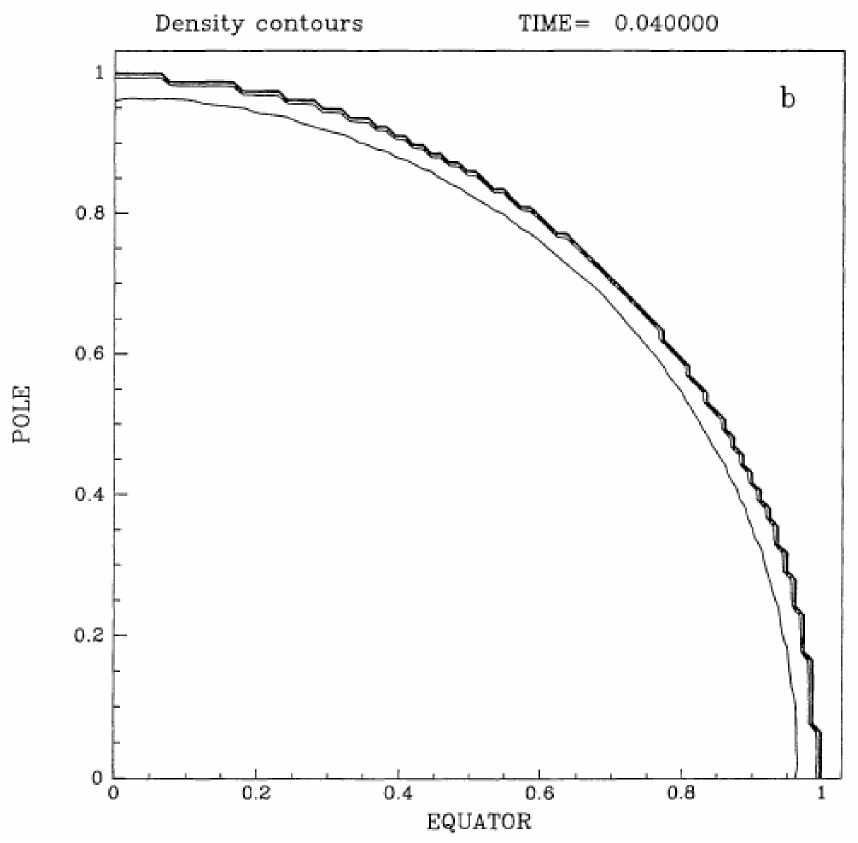


Коллапс быстровращающегося холодного протозвездного облака

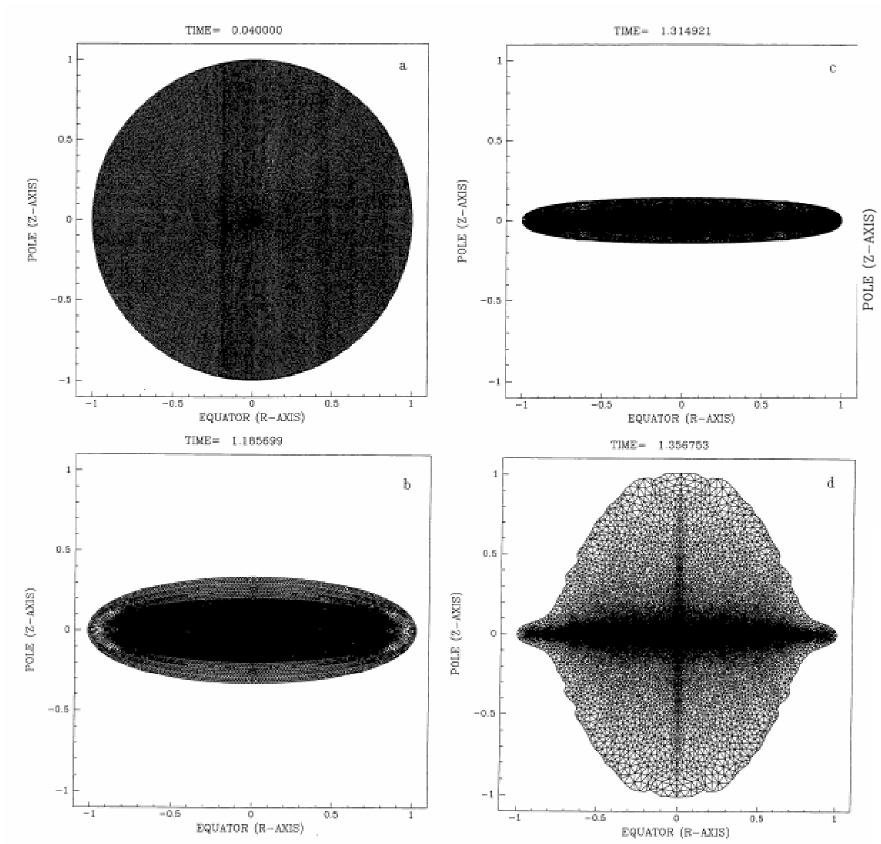
N.V.Ardeljan, G.S.Bisnovatyi-Kogan, K.V.Kosmachevskii, S.G.Moiseenko A&ASS, 1996, 115,573

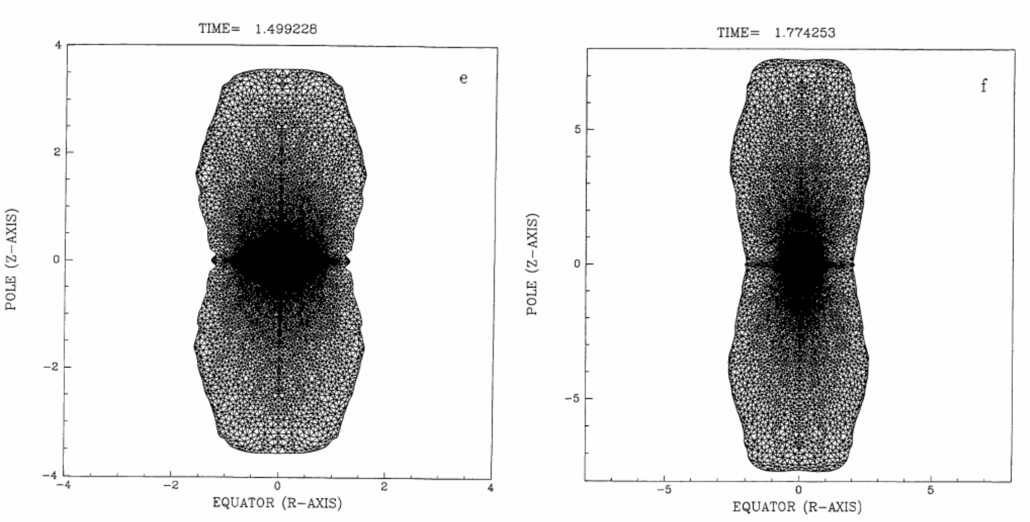
Начальное состояние: $\alpha = E_{\rm in}/|E_{\rm gr}| = 0.00425,\ \beta = E_{\rm rot}/|E_{\rm gr}| = 0.324,$





Коллапс быстровращающегося холодного протозвездного облака





$$\alpha = E_{\rm in}/|E_{\rm gr}| = 0.00425,\ \beta = E_{\rm rot}/|E_{\rm gr}| = 0.324,$$

Сверхновые с коллапсирующим ядром

Сверхновые типа Ib, Ic, II

Взрыв сверхновой – смерть звезды.

Остаток взрыва – нейтронная звезда, черная дыра.

Коллапс железного ядра. Нейтронизация.

Нейтринное излучение.

Энергия взрыва сверхновой ~ 10(51) эрг(!)

Объяснение механизма взрыва сверхновых с коллапсирующим ядром – одна из актуальных задач астрофизики.

Magnetorotational mechanism for the supernova explosion Bisnovatyi-Kogan (1970)(original article was submitted: September 3, 1969)

Amplification of magnetic fields due to differential rotation, angular momentum transfer by magnetic field. Part of the rotational energy is transformed to the energy of explosion

First 2D calculations: LeBlanck&Wilson (1970))(original article was submitted: September 25, 1969) ->too large initial magnetic fields. $E_{mag0} \sim E_{grav} \Rightarrow$ axial jet

Bisnovatyi-Kogan et al 1976, Meier et al. 1976, Ardeljan et al.1979, Mueller & Hillebrandt 1979, Symbalisty 1984, Ardeljan et al. 2000, Wheeler et al. 2002, 2005, Yamada & Sawai 2004, Kotake et al. 2004, 2005, 2006, Burrows et al.2007, Sawai, Kotake, Yamada 2008…

It is popular now!

The realistic values of the magnetic field are: $E_{mag} << E_{grav}$ ($E_{mag}/E_{grav} = 10^{-8}$-$10^{-12}$)

Small initial magnetic field -is the main difficulty for the numerical simulations.

The hydrodynamic time scale is much smaller than the magnetic field amplification time scale (*if magnetorotational instability is neglected*).

Explicit difference schemes can not be applied. (CFL restriction on the time-step).

Implicit schemes should be used.

Basic equations: MHD +self-gravitation, infinite conductivity:

$$\begin{cases} \frac{dx}{dt} = \mathbf{u}, \frac{d\rho}{dt} + \rho \mathrm{div}\mathbf{u} = 0, \\ \rho \frac{du}{dt} = -\mathrm{grad}\left(p + \frac{\mathbf{H}\cdot\mathbf{H}}{8\pi} \right) + \frac{1}{4\pi}\mathrm{div}(\mathbf{H}\otimes\mathbf{H}) - \rho \mathrm{grad}\Phi \\ \rho \frac{d\varepsilon}{dt} + p\mathrm{div}\mathbf{u} + \rho F(\rho,T) = 0, p = P(\rho,T), \varepsilon = \mathrm{E}(\rho,T), \\ \Delta\Phi = 4\pi \mathrm{G}\rho, \\ \rho \frac{d}{dt}\left(\frac{\mathbf{H}}{\rho} \right) = \mathbf{H}\cdot\nabla\mathbf{u}. \end{cases}$$

Additional condition div**H**=0

Axis symmetry ($\frac{\partial}{\partial\phi} = 0$) and equatorial symmetry (z=0) are supposed.

Notations:

$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla, \mathbf{x} = (\mathrm{r},\varphi,\mathrm{z}), \mathbf{u}$ – velocity, ρ – density, p – pressure,

$\mathbf{H}$ – magnetic field, Φ – gravitational potential, ε – internal energy,

G – gravitational constant.

Boundary conditions

Axial symmetry

$$r = 0: u_r = u_\phi = H_r = H_\phi = \mathrm{rot}_\mathrm{r}\mathbf{H} = \mathrm{rot}_\phi\mathbf{H} = 0,$$

Equatorial symmetry

$$u_z = H_z = 0,$$

Quadrupole field

$z = 0:$ or

$$u_z = \frac{\partial B_z}{\partial z} = 0,$$

Dipole field

Outer boundary: $P = \rho = T = H_\phi = 0, \mathbf{H}_\mathrm{poloidal} = \mathbf{H}_\mathrm{q}$

(from Biot-Savart law)

Presupernova Core Collapse

Equations of state take into account degeneracy of electrons and neutrons, relativity for the electrons, nuclear transitions and nuclear interactions. Temperature effects were taken into account approximately by the addition of radiation pressure and an ideal gas

.

Neutrino losses were taken into account in the energy equations.

A cool white dwarf was considered at the stability limit with a mass equal to the Chandrasekhar limit.

To obtain the collapse we increase the density at each point by 20% and we also impart uniform rotation on it.

Equations of state (approximation of tables)

$$P \equiv P(\rho,T) = P_0(\rho) + \Re T\rho + \frac{T^4\sigma}{3}$$

$$P_0(\rho) = \begin{cases} P_0^{(1)} = b_1\rho^{5/3}(1+c_1\rho^{1/3}),\ for\rho \le \rho_1 \\ P_0^{(k)} = a\cdot 10^{b_k(\lg\rho-8.419)^{c_k}}\ for\rho_{(k-1)} \le \rho \le \rho_k, k=\overline{2,6}. \end{cases} \quad \varepsilon_0(\rho) = \int_0^\rho \frac{P_0(x)}{x^2}dx.$$

$$\varepsilon = \varepsilon(\rho,T) = \varepsilon_0(\rho) + \frac{3}{2}\Re T + \frac{\sigma T^4}{\rho} + \varepsilon_{Fe}(\rho,T),\ \varepsilon_{Fe}(\rho,T) = \frac{E_{b,Fe}}{A_{m_p}}\left(\frac{T-T_{0,Fe}}{T_{1,Fe}-T_{0,Fe}}\right),$$

Fe –dis-sociation

Neutrino losses: **URCA processes, pair annihilation, photo production of neutrino, plasma neutrino**

URCA: $f(\rho,\overline{T}) = 1.3\cdot 10^9\chi(\overline{T})/[1+(7.1\cdot 10^{-5}\rho/\overline{T}^3)^{2/5}]erg\cdot g^{-1}\cdot c^{-1}$

$$\lambda(T) = \begin{cases} 1, \overline{T} < 7, \\ 664.31 + 51.024(\overline{T}-20), 7 \le \overline{T} \le 20, \\ 664.31, \overline{T} > 20, \end{cases} \qquad \overline{T} = T\cdot 10^{-9}.$$

Approximation of tables from Ivanova, Imshennik, Nadyozhin,1969

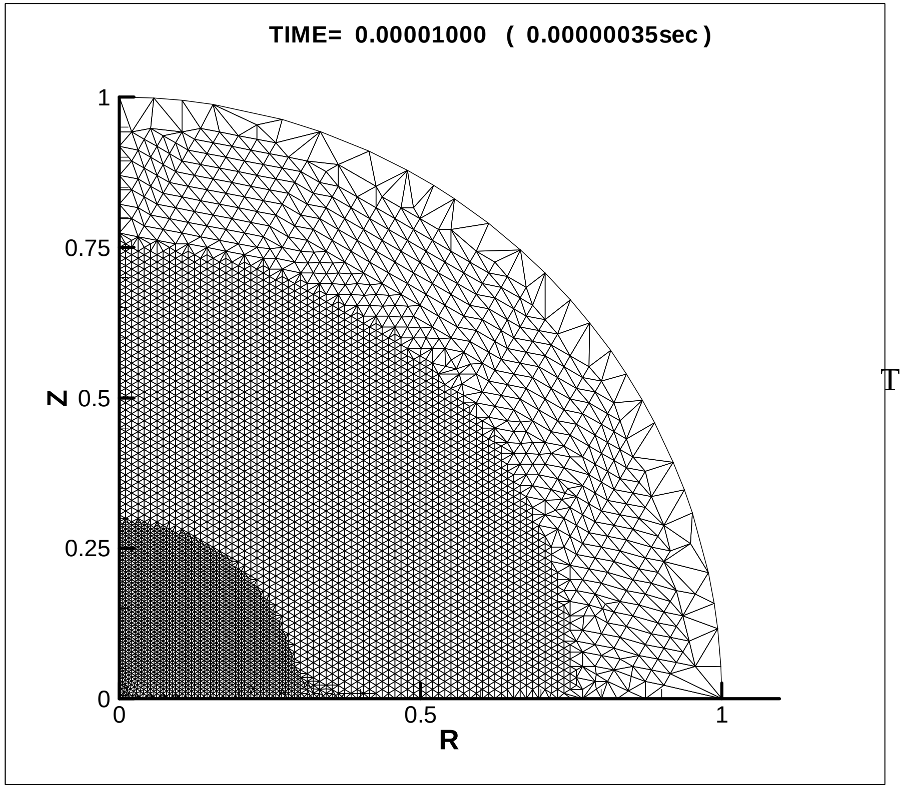
$$F(\rho,T) = f(\rho,T)e^{-\tau_\nu}$$

neutrino diffusion

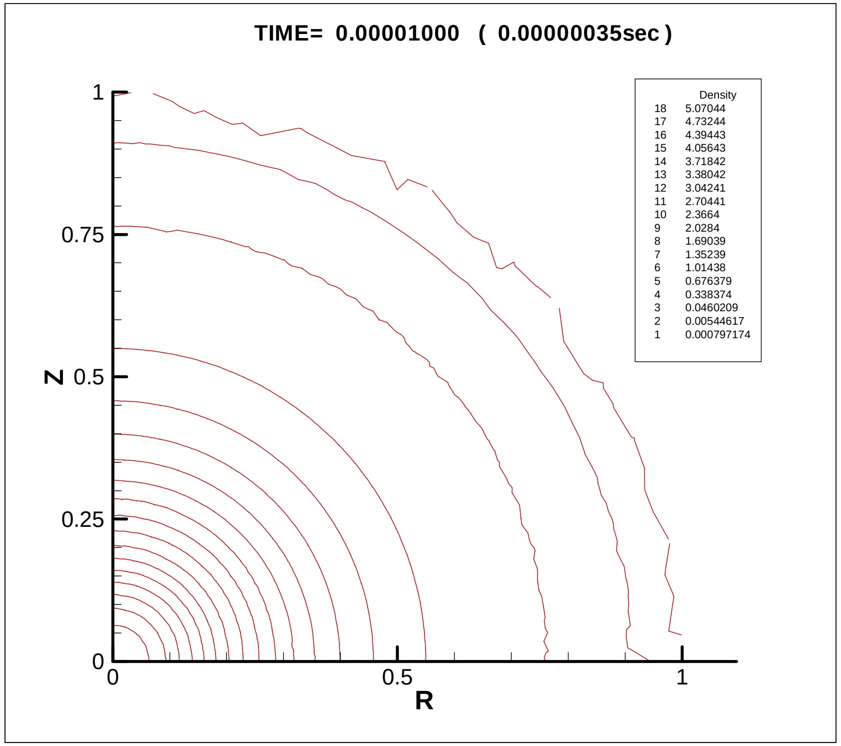
Initial state

$M = 1.2042 \cdot M_{sun}$, spherically symmetrical stationary state, initial angular velocity 2.519 (1/sec)

Initial temperature distribution $T = \delta\rho^{2/3}$



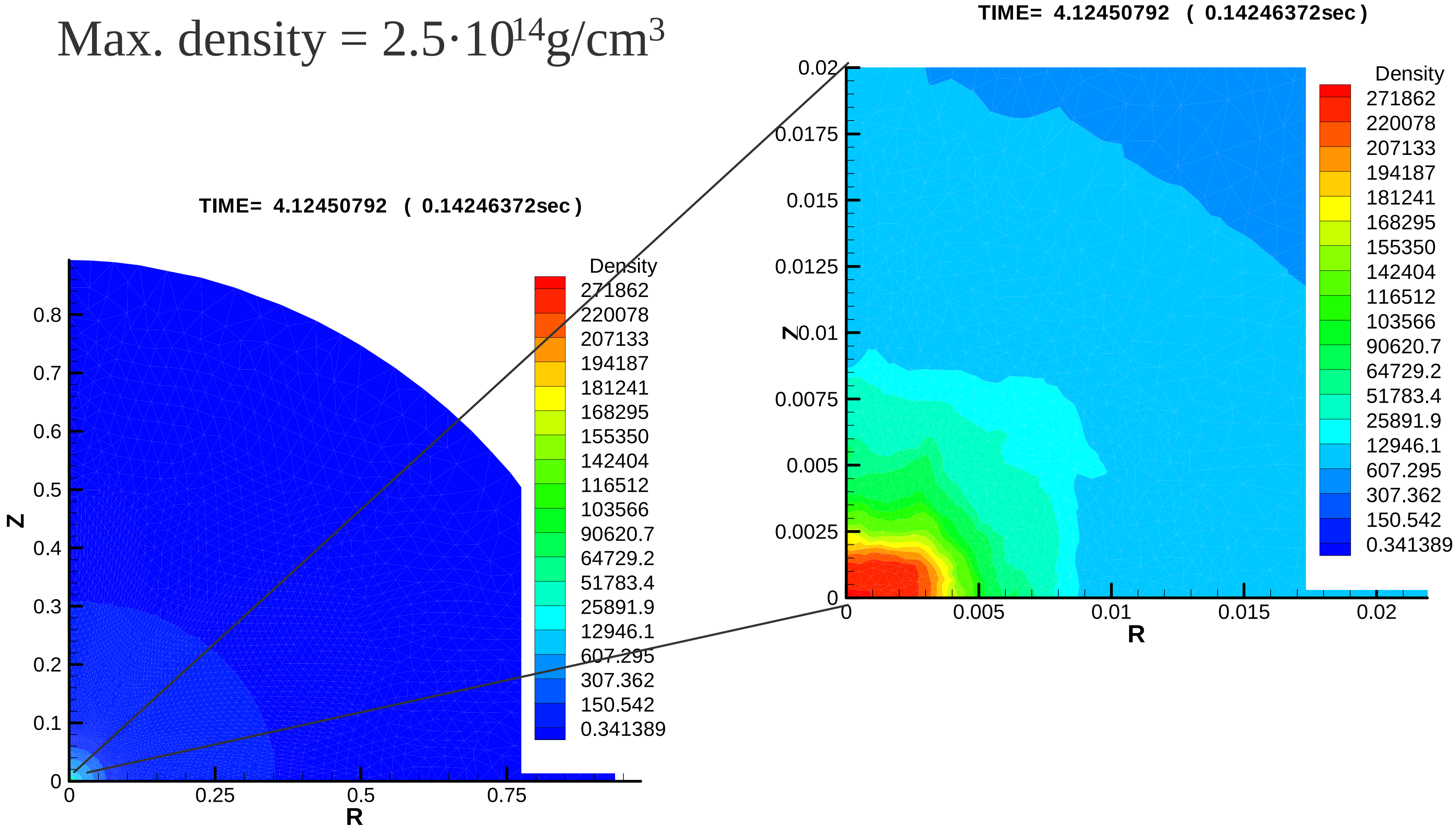
T



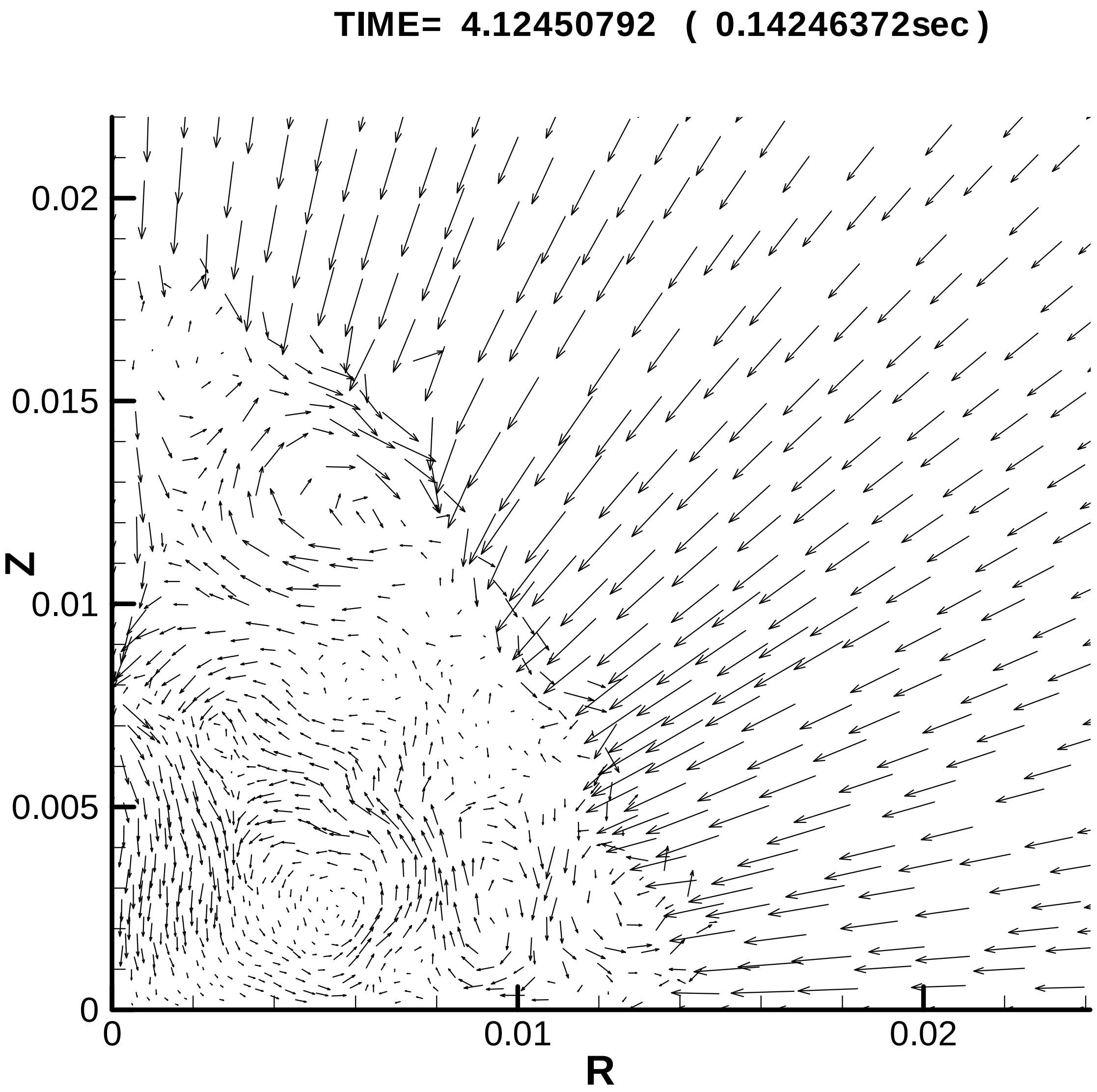
$$\frac{E^{rot}}{E^{grav}} = 0.571\% \qquad \frac{E^{\text{int}}}{E^{grav}} = 72.7\%$$

Maximal compression state

Max. density = $2.5\cdot10^{14}$ g/cm^3

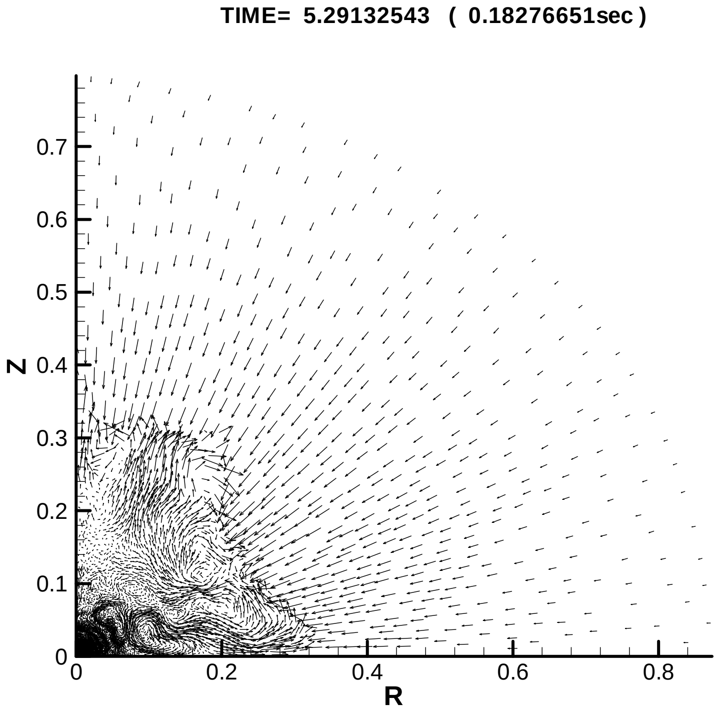


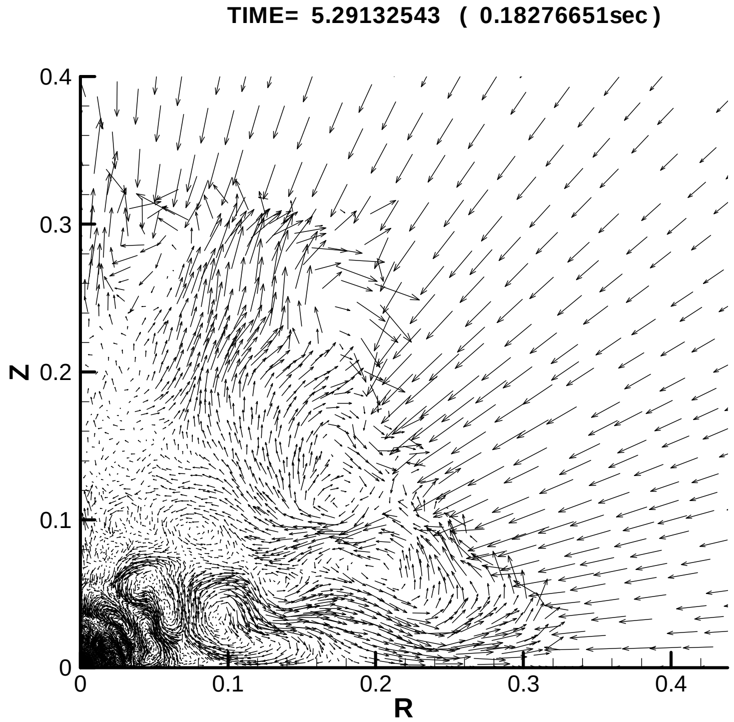
Neutron star formation in the center and formation of the shock wave



«0.01»~10km

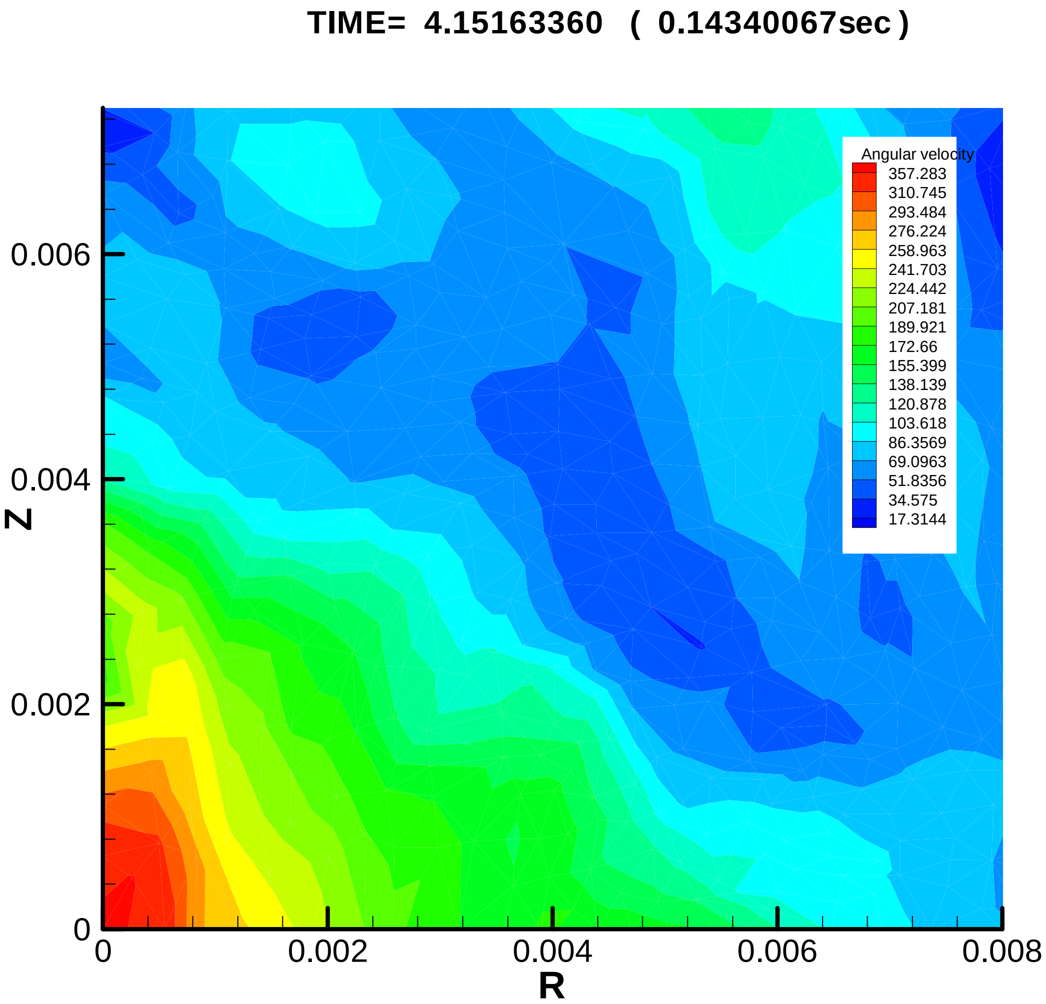
Mixing



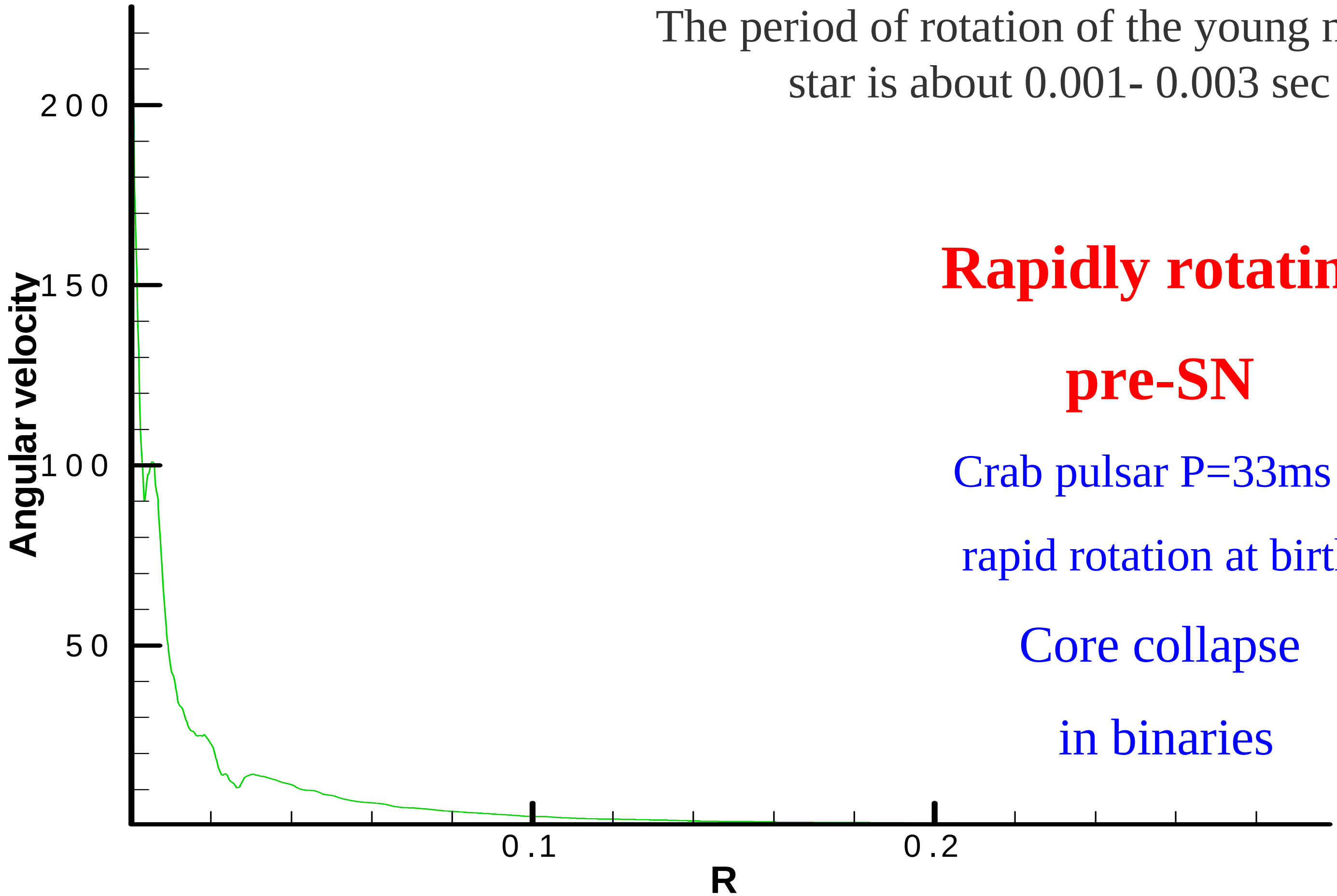


Bounce shock wave does not produce SN explosion :(

Angular velocity (central part of the computational domain). Rotation is differential.



Distribution of the angular velocity



The period of rotation of the young neutron star is about 0.001- 0.003 sec

Rapidly rotating

pre-SN

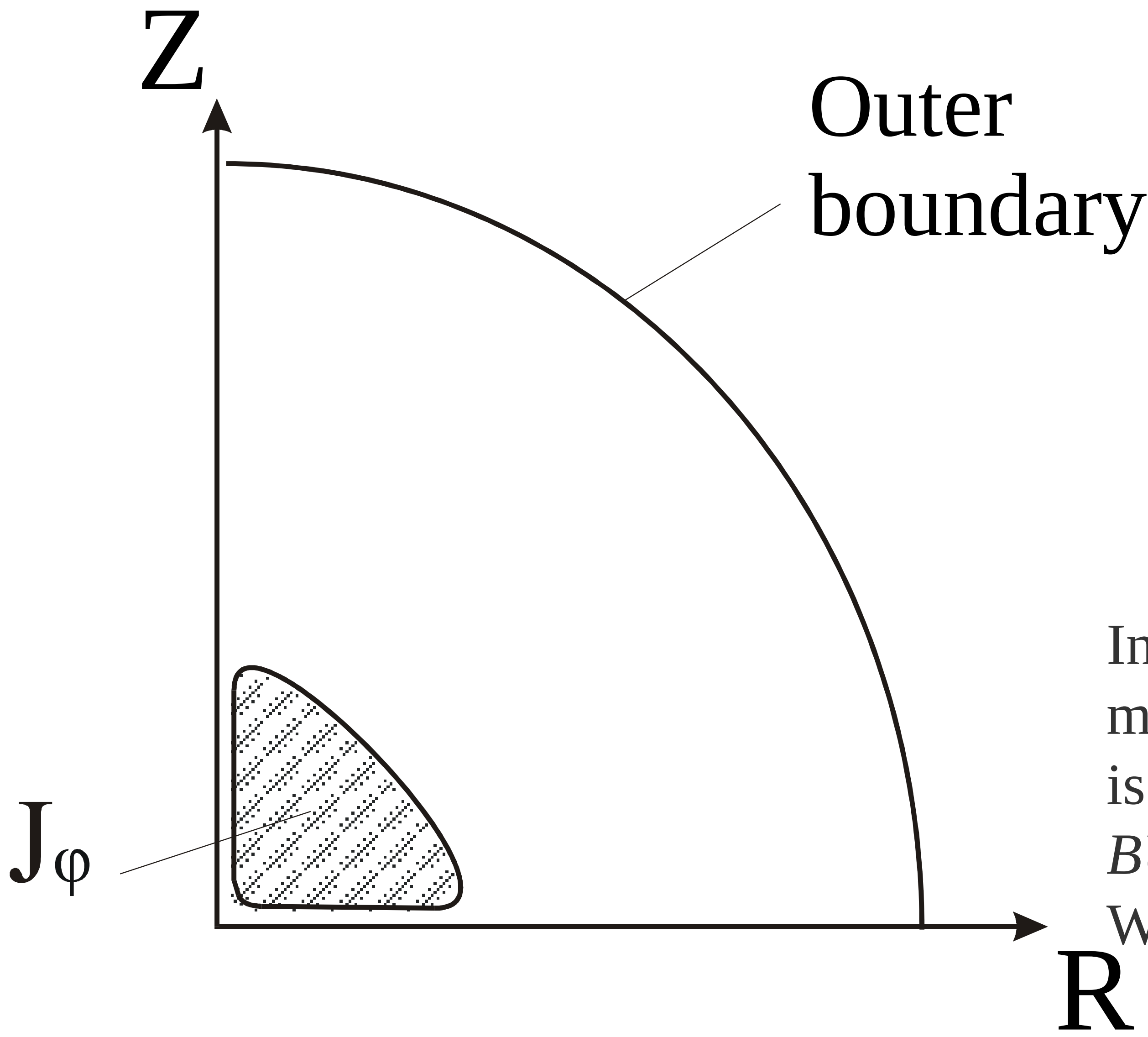
Crab pulsar P=33ms –

rapid rotation at birth

Core collapse

in binaries

Initial magnetic field



$$\mathbf{H} = \frac{1}{c}\int \frac{\mathbf{J}\times\mathbf{R}}{R^3}\,dv,$$

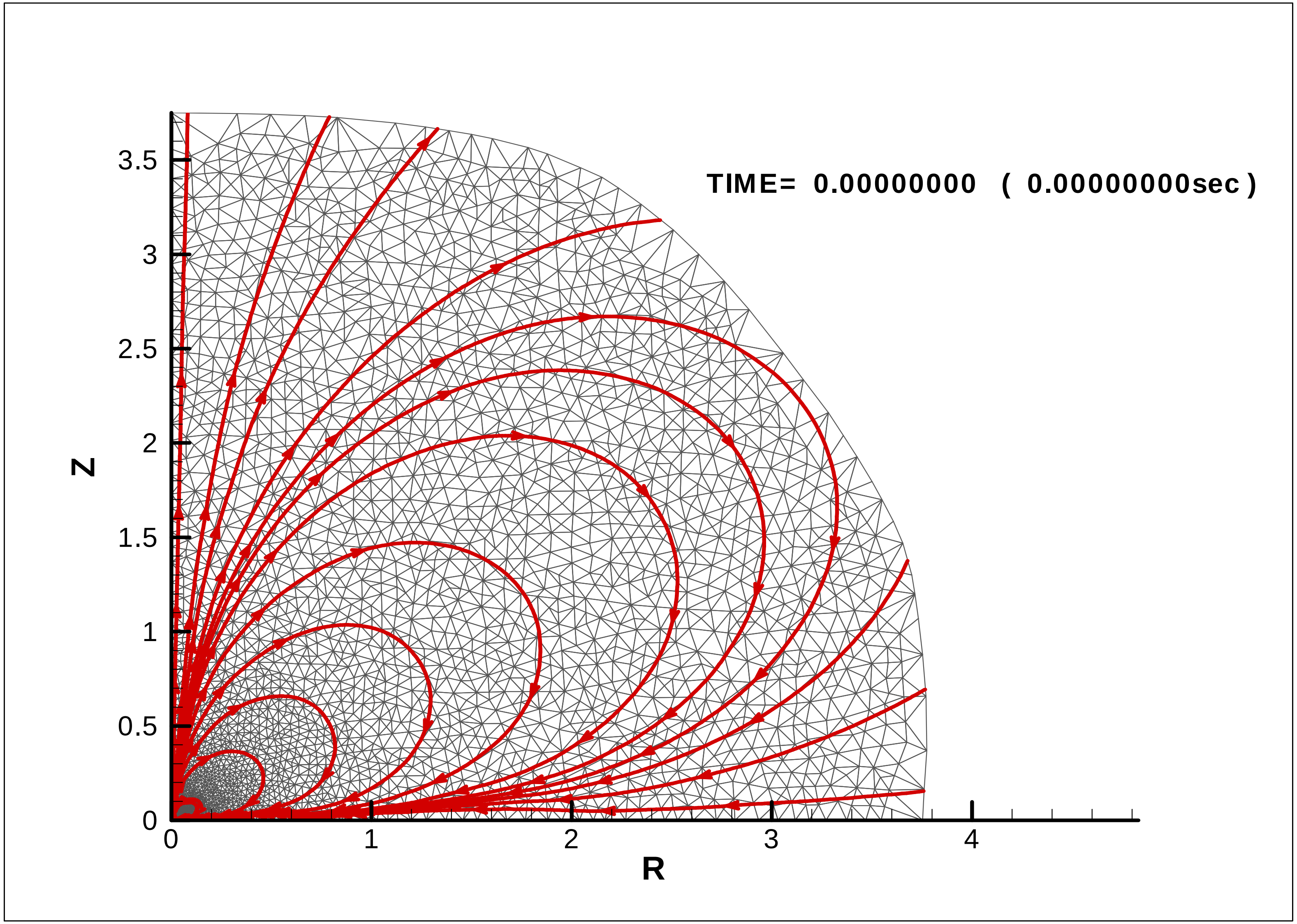
Biot-Savart law

$$J_\phi -> (H_r, H_z)$$

Initial poloidal
magnetic field
is divergence free,
BUT not balanced.
We have to balance it.

Initial magnetic field –quadrupole-like symmetry

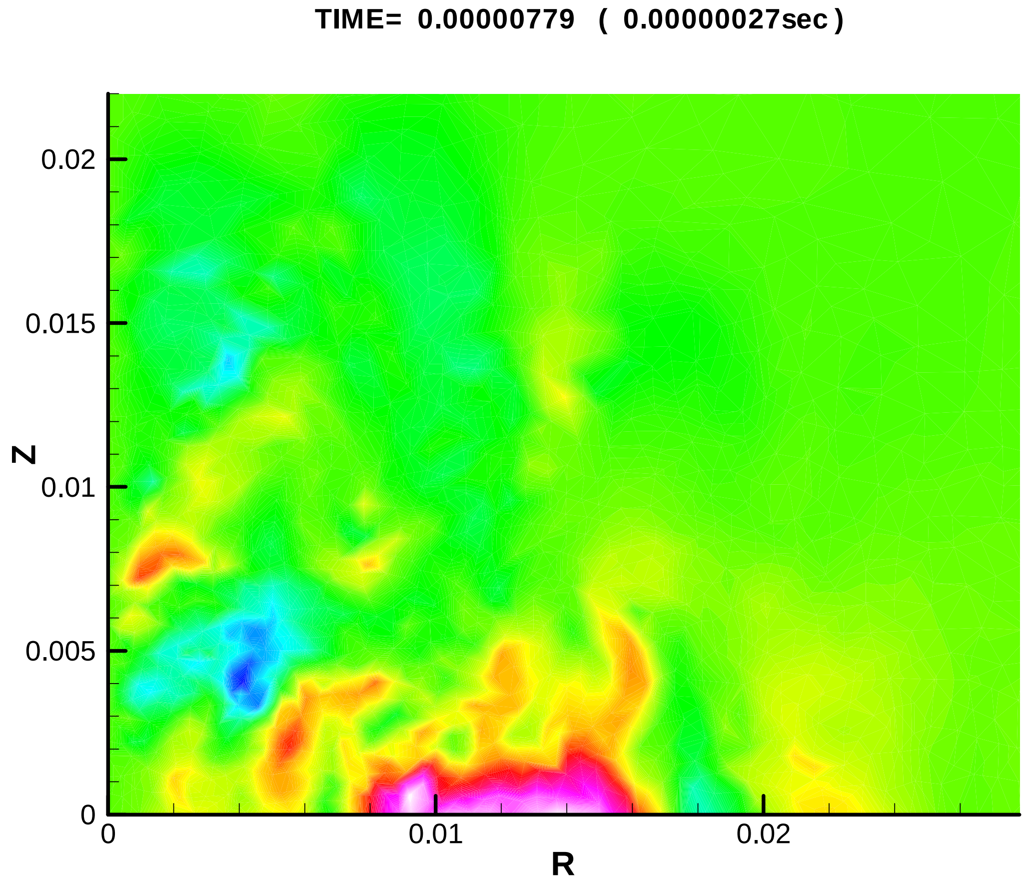
Ardeljan, Bisnovatyi-Kogan, SM, MNRAS 2005, 359, 333



Toroidal magnetic field amplification.

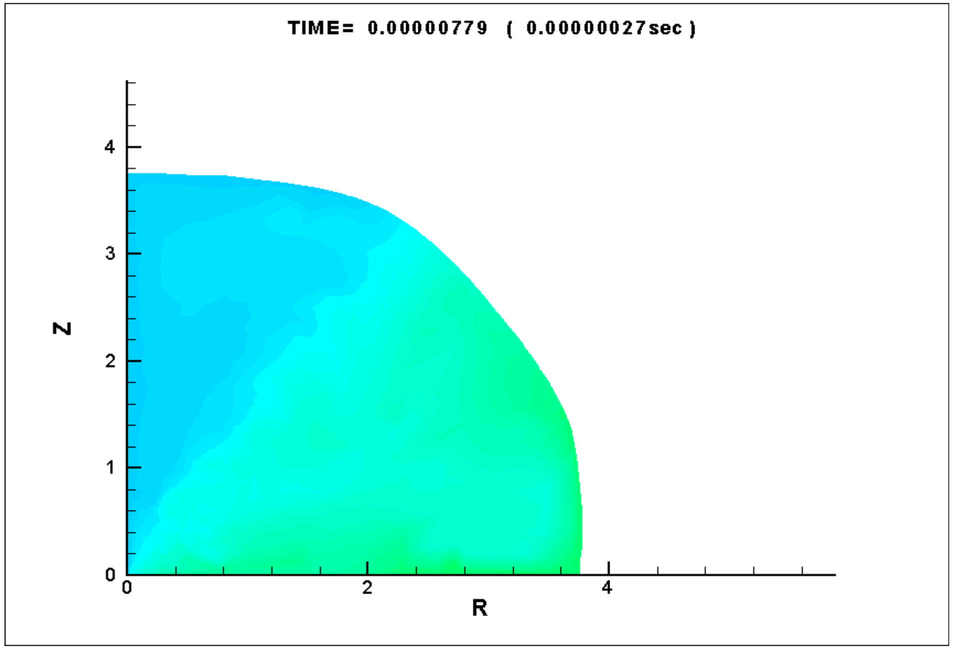
pink – maximum_1 of Hf^2 blue – maximum_2 of Hf^2

Maximal values of Hf=2.5 ·10(16)G

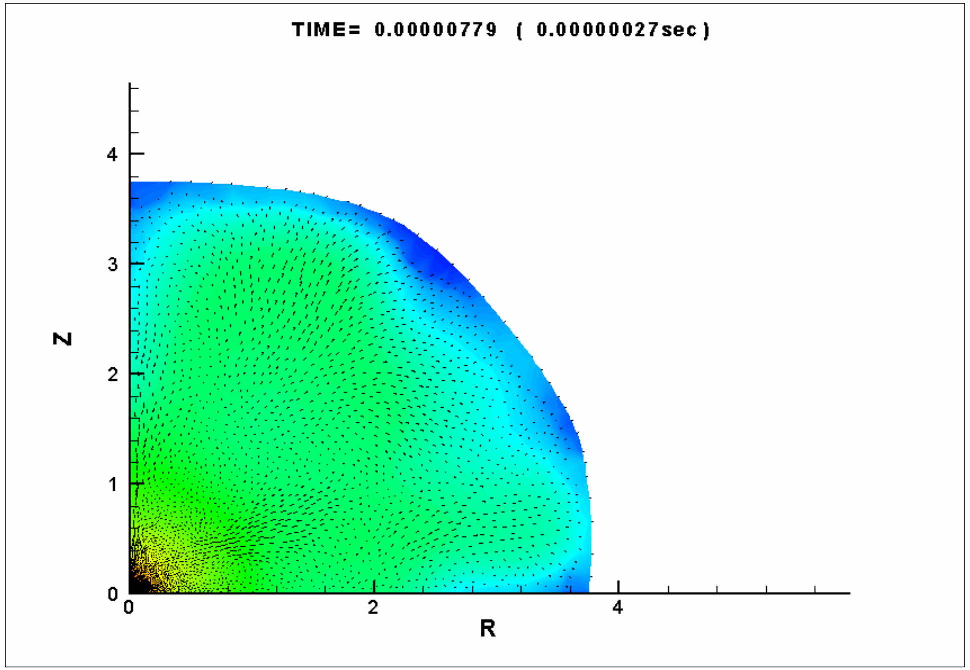


After SN explosion at the border of neutron star H=$2 \cdot 10^{14}$G

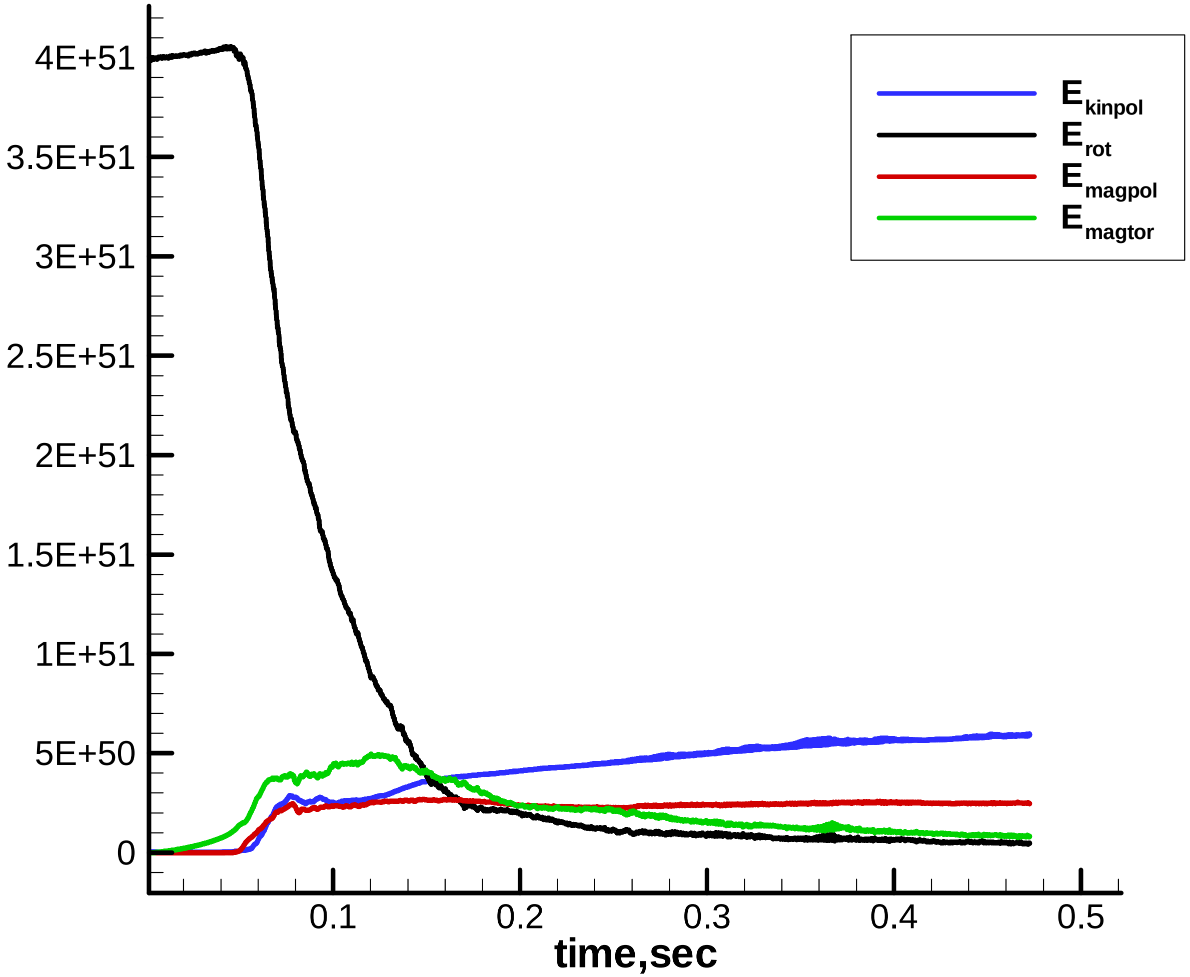
Specific angular momentum rV_{φ}



Temperature and velocity field

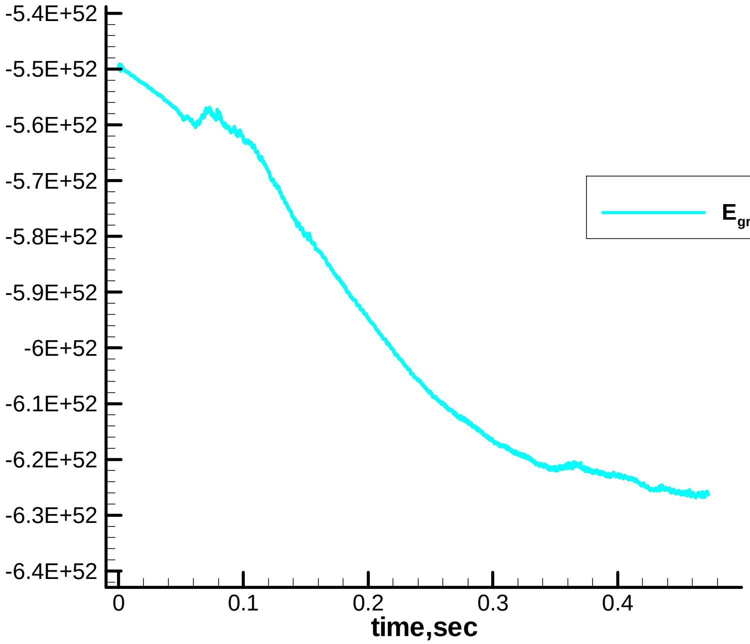


Time evolution of different types of energies

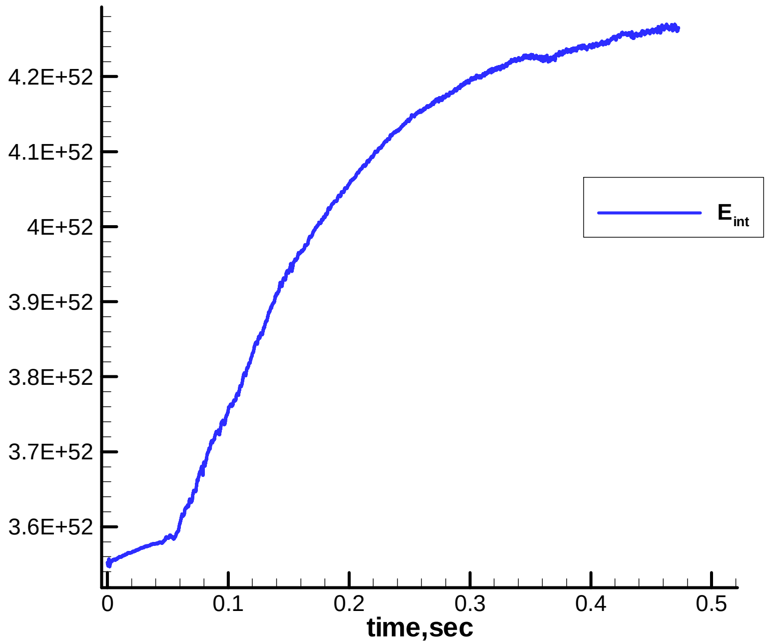


Time evolution of the energies

Gravitational energy

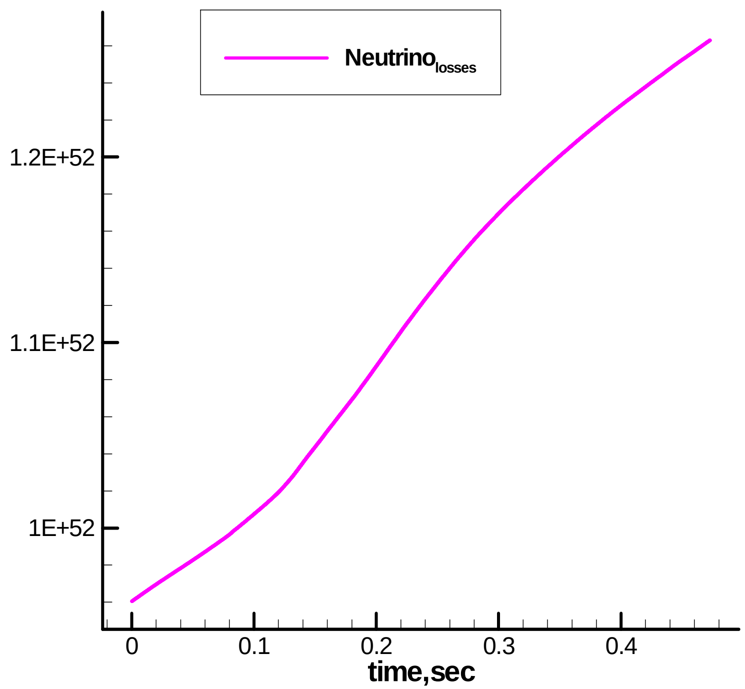


Internal energy

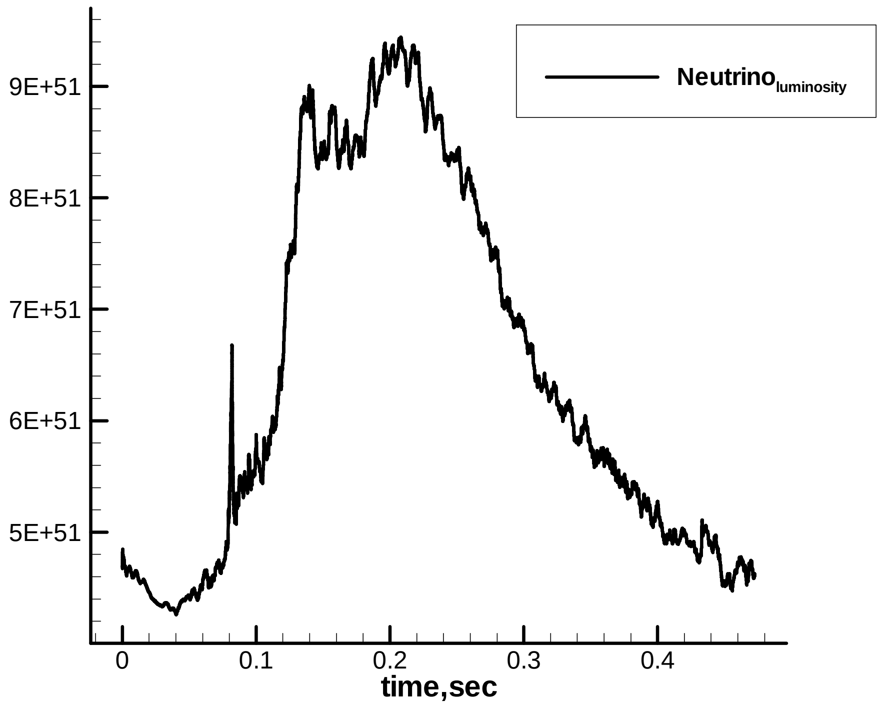


Time evolution of the energies

Neutrino losses (ergs)



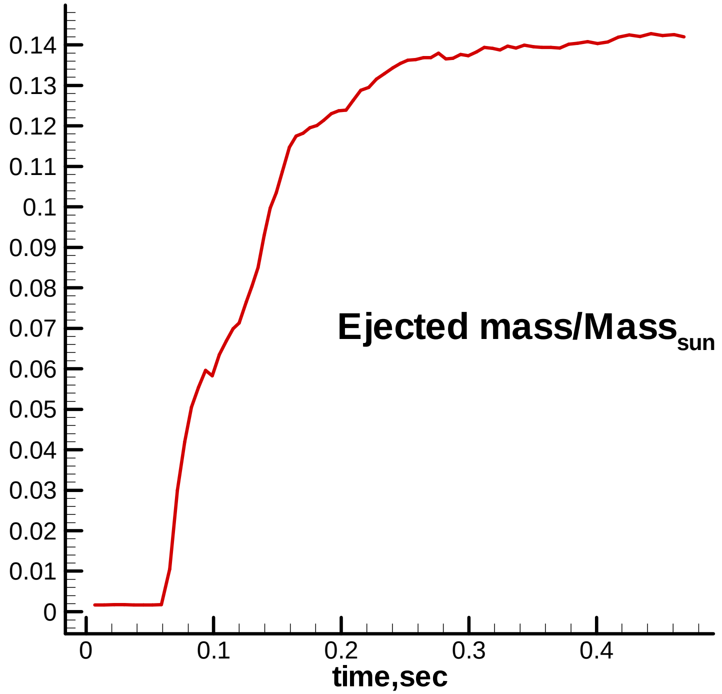
Neutrino luminosity (ergs/sec)

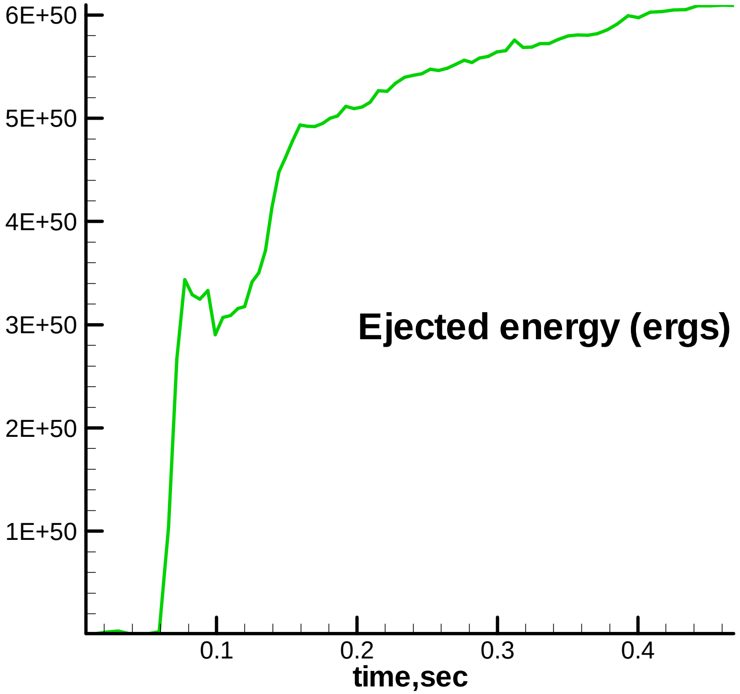


Ejected energy and mass

Ejected energy $0.6 \cdot 10^{51} erg$ Ejected mass $0.14 M_{\odot}$

Particle is considered "ejected" –
if its kinetic energy is greater than its potential energy

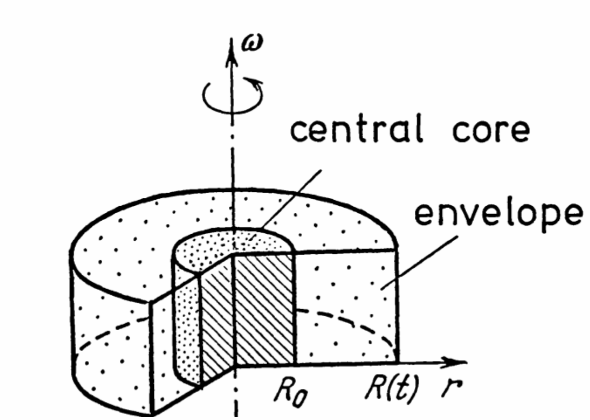




Magnetorotational supernova in 1D

Bisnovaty-Kogan et al. 1976, Ardeljan et al. 1979

$$t_{\text{explosion}} \qquad \frac{1}{\sqrt{\alpha}}, \qquad \left(\alpha = \frac{E_{\text{mag0}}}{E_{\text{grav0}}}\right)$$



Example: $\alpha = 10^{-2} \Rightarrow t_{\text{explosion}} = 10,$

$\alpha = 10^{-12} \Rightarrow t_{\text{explosion}} = 10^{6}$!!!

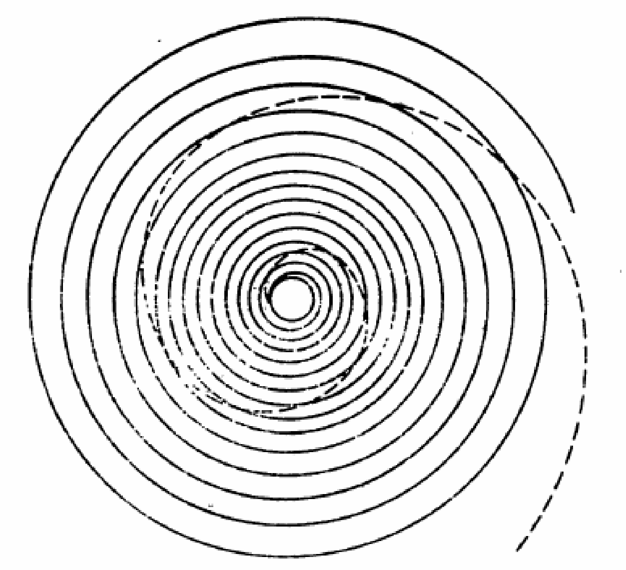
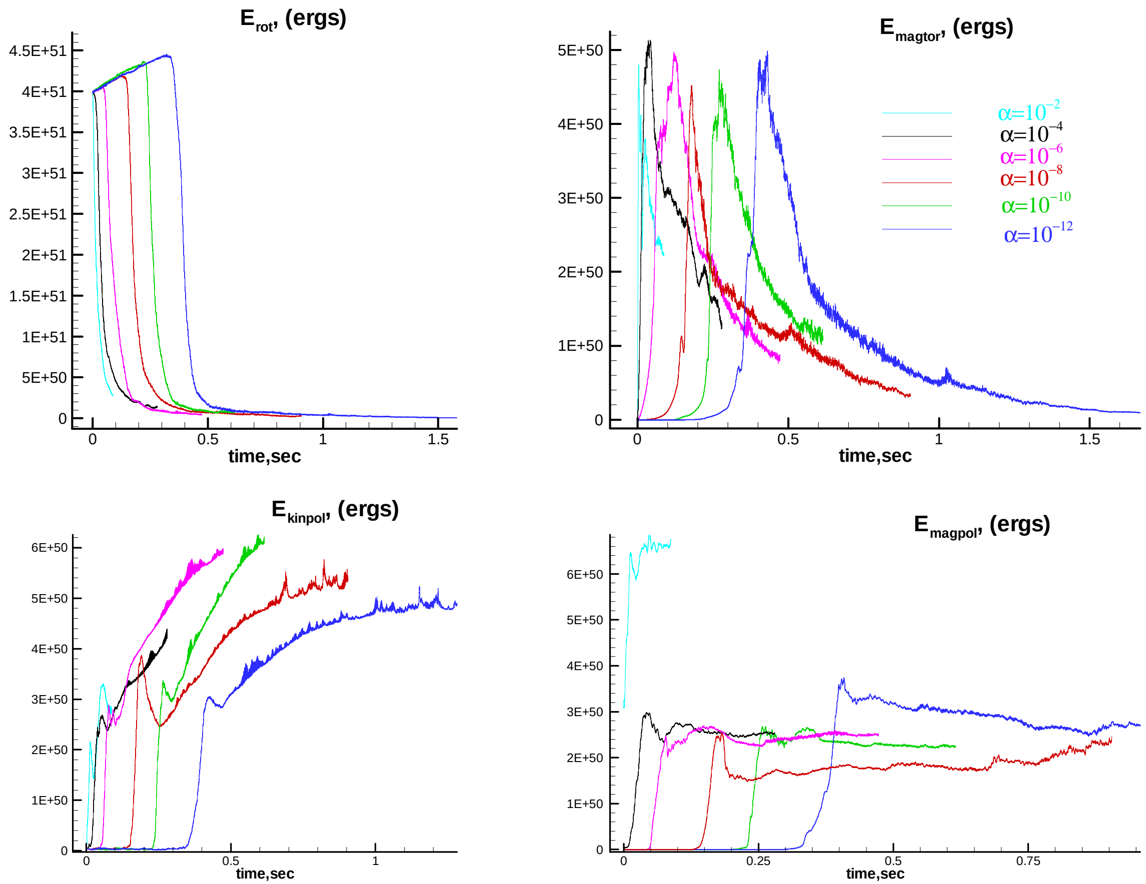


FIG. 3. Shape of a field line in the region near the core at the time $t_\alpha = 7$ for $\alpha = 10^{-2}$ (dashed line) and $\alpha = 10^{-4}$ (solid line).

Magnetorotational explosion for the different $\alpha = \frac{E_{mag0}}{E_{grav0}} = 10^{-2} - 10^{-12}$

Magnetorotational instability$\Rightarrow$ mag. field grows exponentially
(Dungey 1958,Velikhov 1959, Chandrasekhar, Balbus & Hawley 1991,
Spruit 2002, Akiyama et al. 2003…)



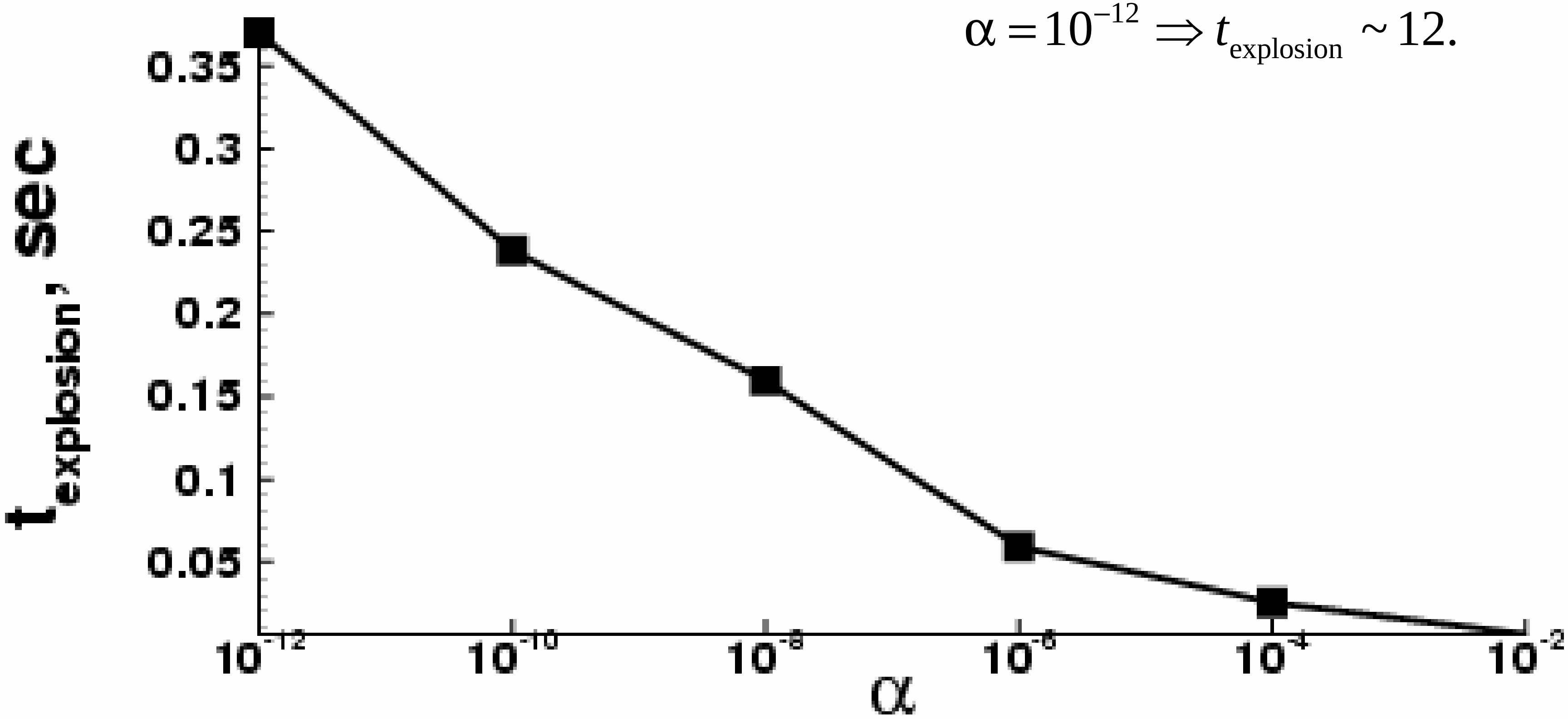
Dependence of the explosion time from $\alpha = \frac{E_{mag0}}{E_{grav0}}$

$t_{explosion} \sim -\log(\alpha)$ (for small α)

Example:

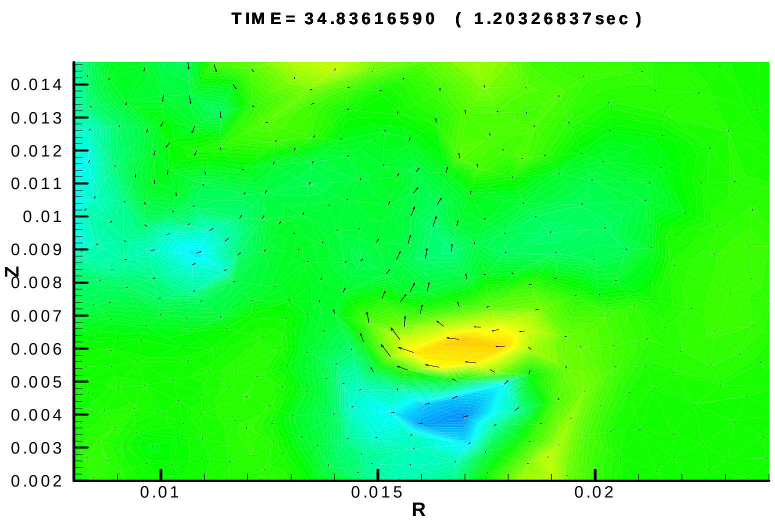
$\alpha = 10^{-6} \Rightarrow t_{explosion} \sim 6,$

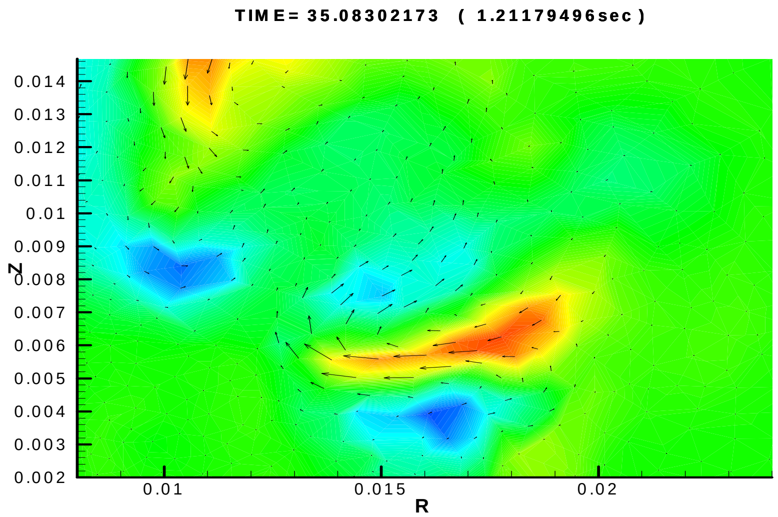
$\alpha = 10^{-12} \Rightarrow t_{explosion} \sim 12.$

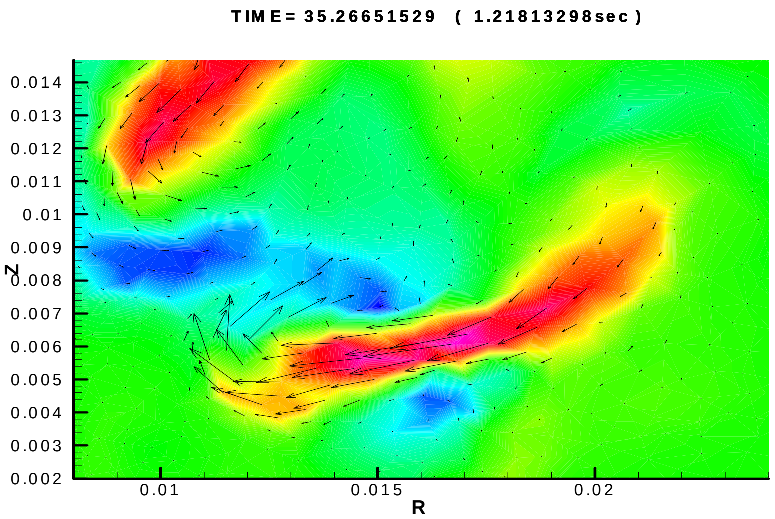


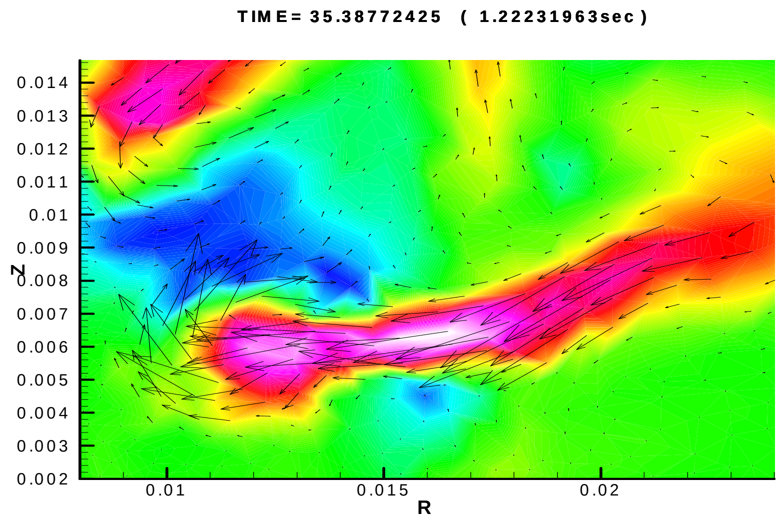
Magnetorotational instability

Central part of the computational domain . Formation of the MRI.









Toy model for MRI in the magnetorotational supernova

$\frac{dH_\varphi}{dt} = H_r\left(r\frac{d\Omega}{dr}\right);$ at the initial stage of the process $H_\varphi < H_\varphi^*$: $H_r\left(r\frac{d\Omega}{dr}\right) \approx \text{const},$

beginning of the MRI => formation of multiple *poloidal* differentially rotating

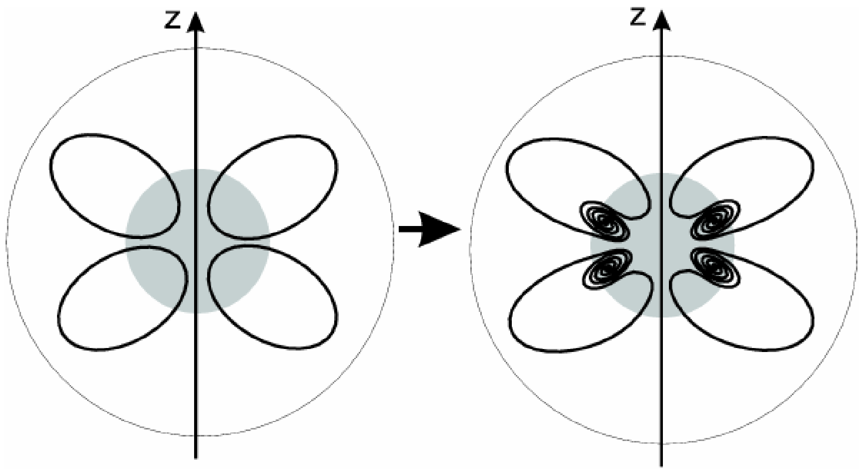
vortexes $\frac{dH_r}{dt} = H_{r0}\left(\frac{d\omega_v}{dl}l\right),$ in general we may approximate: $\left(\frac{d\omega_v}{dl}l\right) \approx \alpha(H_\varphi - H_\varphi^*).$

Assuming for the simplicity that $\left(r\frac{d\Omega}{dr}\right) = A$ is a constant during the first stages of MRI, and taking H_φ^* as a constant we come to the following equation:

$$\frac{d^2}{dt^2}\left(H_\varphi - H_\varphi^*\right) = AH_{r0}\alpha(H_\varphi - H_\varphi^*)$$

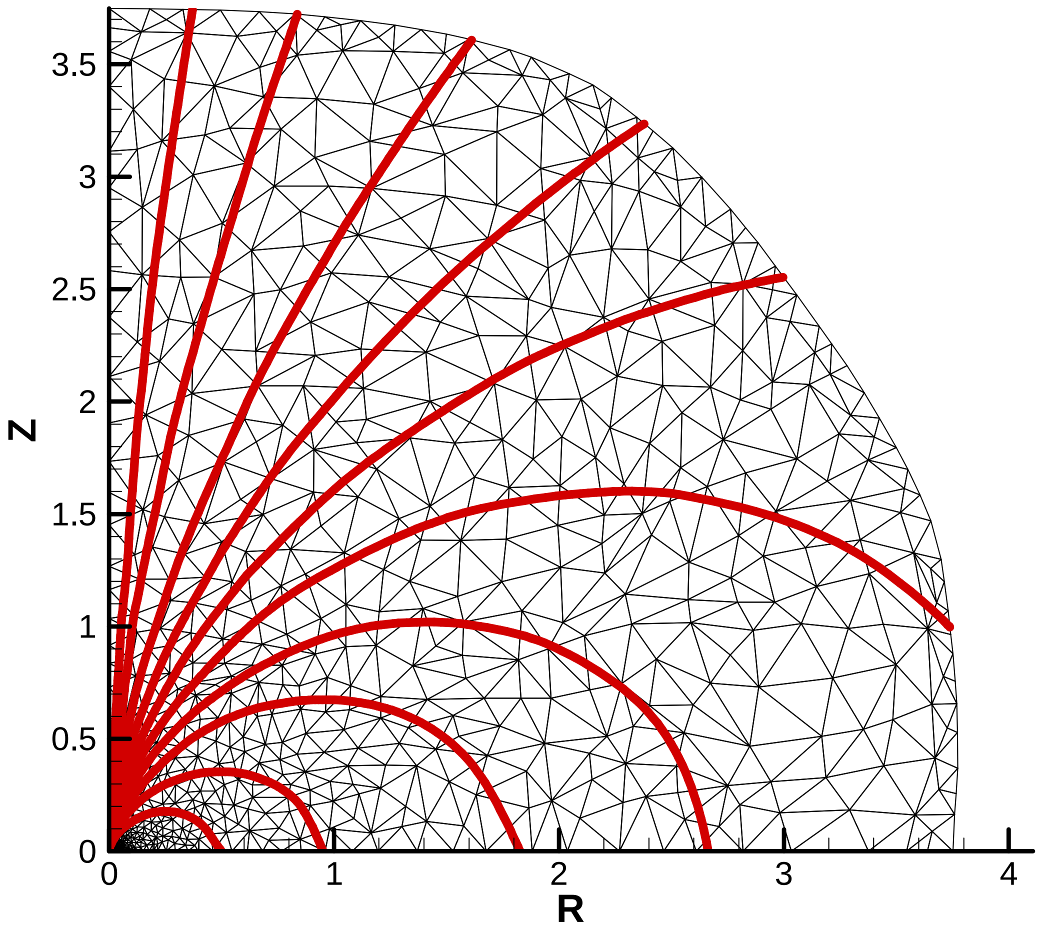
$$\Downarrow$$

$$\begin{cases} H_\varphi = H_\varphi^* + H_{r0}e^{\sqrt{A\alpha H_{r0}}\,(t-t^*)}, \\ H_r = H_{r0} + \frac{H_{r0}^{3/2}\alpha^{1/2}}{\sqrt{A}}\left(e^{\sqrt{A\alpha H_{r0}}\,(t-t^*)} - 1\right). \end{cases}$$

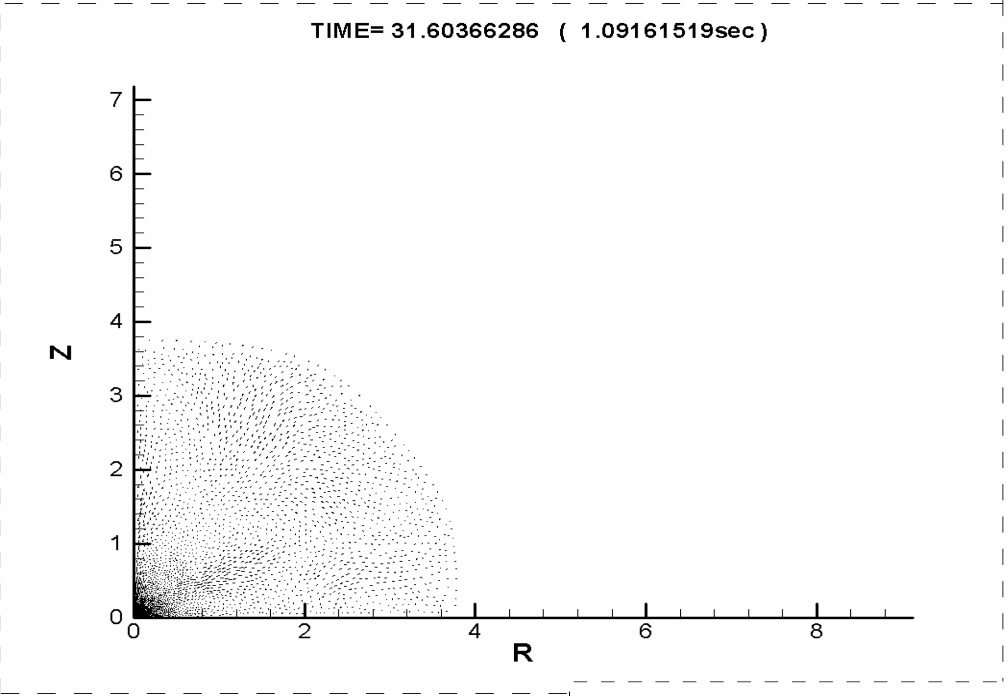


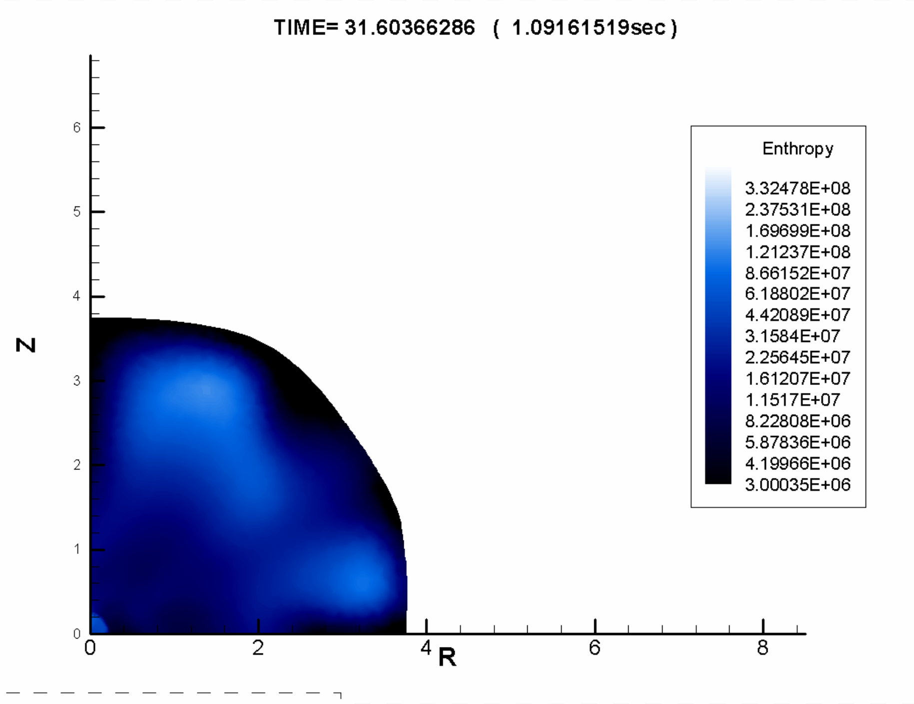
Initial magnetic field – dipole-like symmetry

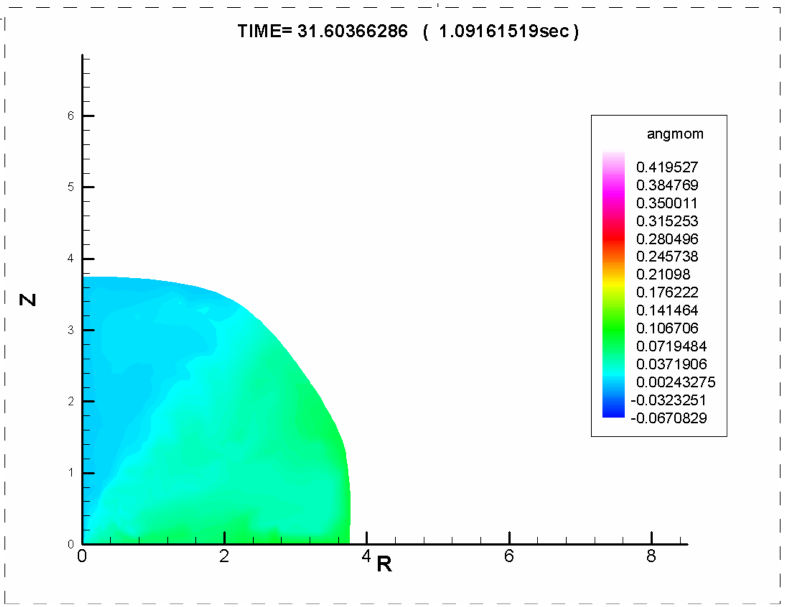
SM., Ardeljan & Bisnovatyi-Kogan MNRAS 2006, 370, 501



Magnetorotational explosion for the dipole-like magnetic field



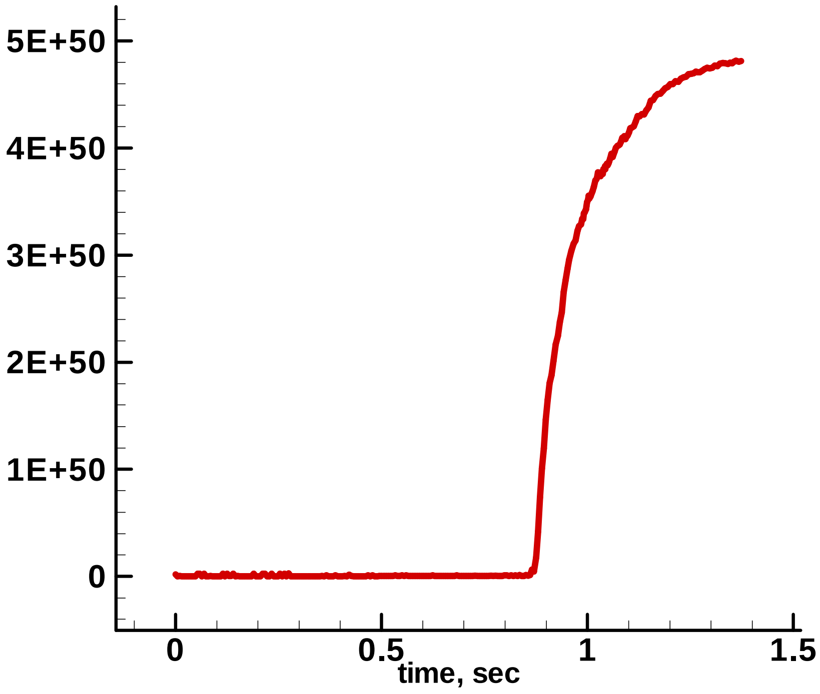


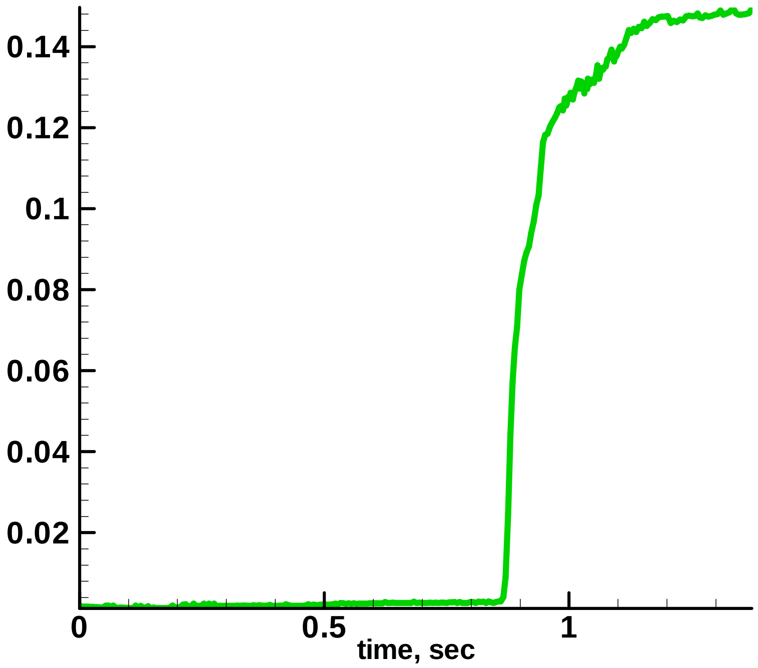


Ejected energy and mass (dipole)

Ejected energy $\approx 0.5 \cdot 10^{51} erg$ Ejected mass $\approx 0.14 M_{\odot}$

Particle is considered "ejected" –
if its kinetic energy is greater than its potential energy





The main difference of bounce shock and neutrino driven mechanisms from magnetorotational supernovae ⇨ magnetic field works like a piston. This MHD piston supports the supernova MHD shock wave for some time.

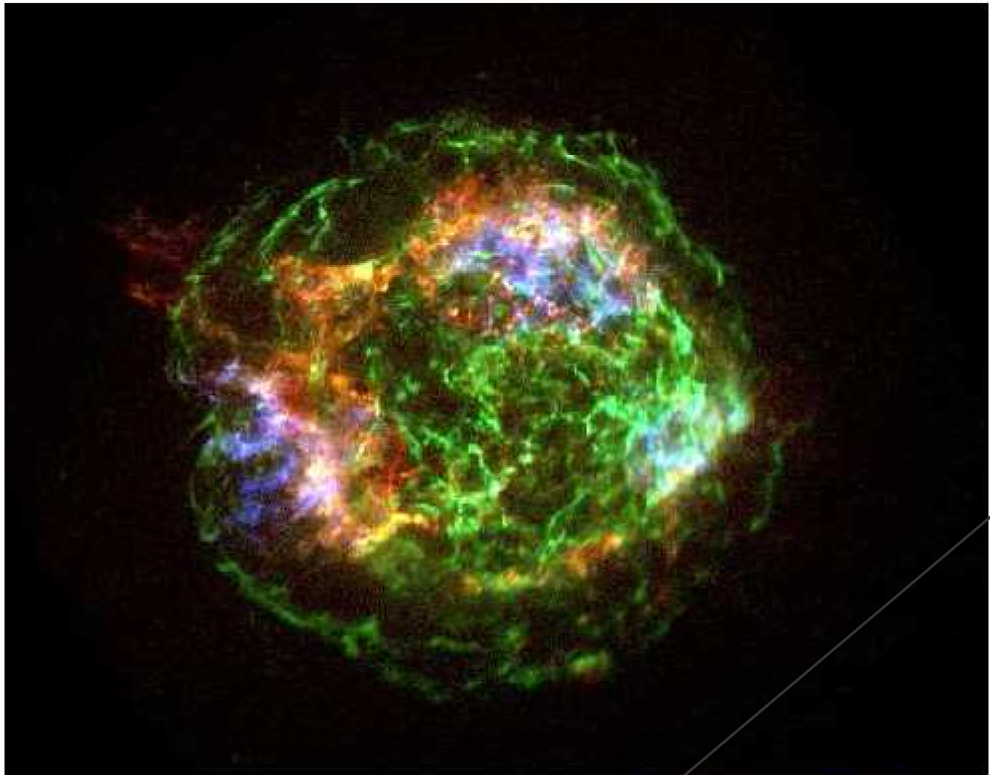
3D features of MR supernova simulations

•Lagrangian grid (tetrahedrons) method requires frequent grid reconstruction near protoneutron star surface (due to differential rotation) => violation of the solution.
•Unstructured grid (Dirichlet sells) – construction of 3D grid of Diriclet cells is expensive.
•SPH (Smooth Particle Hydrodynamics) – poor spatial accuracy, concentration of particles near gravitational center.

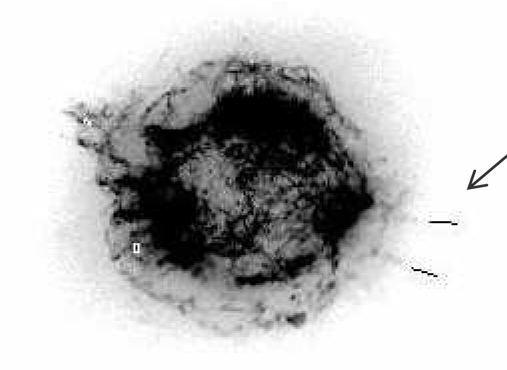
Optimal :
Eulerian scheme with Automatic Mesh Refinement : Approximate MHD Riemann solvers (problems with degeneration of eigenvectors and eigenvalues)
Finite-difference schemes (e.g. central numerical schemes *Kurganov&Tadmor*, *Ziegler*).

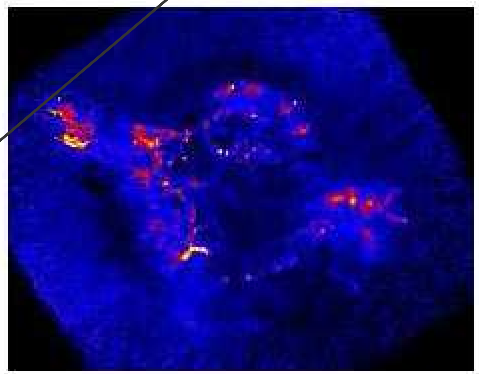
Cassiopea A- supernova with jets-an example of the magnetorotational supernova

(Hwang et al. ApJL, 2004, 615, L117)



1 million seconds Chandra servey of Cas A.
Second jet was found.





Conclusions

- Magnetorotational mechanism (MRM) produces enough energy for the core collapse supernova.
- The MRM is weakly sensitive to the neutrino cooling mechanism.
- MR supernova shape depends on the configuration of the magnetic field and is always asymmetrical.
- MRI developes in MR supernova explosion.
- One sided jets and rapidly moving pulsars can appear due to MR supernovae.
- 3D simulations of MR supernova are necessary.

Спасибо за внимание!