

Introduction: A new model is presented for the coupled evolution of climate and rotation, as applied to Mars. It has long been appreciated that changes in the orbital and rotational geometry of Mars will influence the seasonal and latitudinal pattern of insolation [1-5], and this will likely dominate climatic fluctuations on time scales of 10^5 to 10^7 years [6-9]. Equally important, but less widely appreciated, is the influence climatic change can have on rotational dynamics. The primary means by which climate influences rotation is via its influence on transport of mass (volatiles and dust) into and out of the polar regions. Many important issues remain unresolved: What are the ages of the polar caps? What climatic periods are recorded in the polar layered deposits? What is the long term obliquity history?

Rotational Influence on Climate: The orientation of the rotation axis can be specified by a unit vector \mathbf{s} . Orientation of the orbit is characterized by two unit vectors: the orbit normal \mathbf{n} determines the plane in which the orbit lies, and the apsidal vector \mathbf{a} (pointing from perihelion toward the Sun) determines orientation of the orbit within that plane. Spin axis orientation is determined by projections of \mathbf{s} onto \mathbf{a} and \mathbf{n} . Obliquity $\epsilon = \cos^{-1}(\mathbf{s} \cdot \mathbf{n})$ determines the amplitude of the dominant seasonal temperature cycle. The spatio-temporal pattern of obliquity driven insolation has symmetry such that reflection in the equatorial plane and time translation by half a year are inverse operations. The relative phase of perihelion and summer solstice is determined by the angle $\phi = \cos^{-1}(\mathbf{a} \cdot (\mathbf{s} \times \mathbf{n}))$. In the low obliquity limit, the insolation pattern due to this effect has time reflection symmetry about the solstices.

If Mars were the Sun's only planet, its orbit would remain fixed, and the spin axis \mathbf{s} would precess at a uniform angular rate on a circular cone centered on \mathbf{n} . Obliquity ϵ would remain constant, and the perihelion phase angle ϕ would circulate at a fixed rate. Insolation patterns would change from year to year, but in a perfectly periodic fashion. However, due to gravitational interaction and angular momentum exchange with other planets, the orbit of Mars changes on a variety of time scales [10]. As a result, both the obliquity and the perihelion phase rate fluctuate considerably.

During periods of low obliquity, the polar regions will be especially cold, and will accumulate additional mass, in the form of H_2O and CO_2 ice plus dust. During times of intermediate obliquity, the polar volatiles

will tend to disperse, and in epochs of high obliquity, there may be equatorial ice and dust accumulations [11].

Details of how the Mars climate system responds to orbital and rotational forcing are not well known. For simplicity, assume that the polar ice cap mass depends only on obliquity. In particular, suppose that there is a total ice mass of $m_t = 3 \times 10^{18}$ kg, and that at low obliquity ($\epsilon < \epsilon_1$) it all resides in the polar caps, at high obliquity ($\epsilon > \epsilon_2$) it all resides in equatorial ring, and at intermediate obliquity it is divided between the two locations. The equilibrium mass $m_e(\epsilon)$ of the polar caps depends only on current obliquity, but the climate system kinematics dictate the rate of change of the actual cap mass m_c

$$\frac{dm_c}{dt} = -\left(\frac{m_c - m_e}{\tau_c}\right)$$

All of the climate system dynamics is subsumed into a single relaxation time τ_c . If this climate adjustment time is short, compared to obliquity time scales (10^5 - 10^6 years), the ice cap mass distribution will remain close to equilibrium.

Climatic Influence on Rotation: Changes in the surface mass distribution of Mars, driven by climatic processes, can influence the rotational state.

The rate of spin axis precession depends on the oblateness (departure from spherical symmetry) of the mass distribution. To the extent that climate change can modulate the gravitational oblateness, it will influence the precession. Thus, it seems likely that climatic change in general, and obliquity-driven climate change in particular, will influence oblateness. This, in turn, influences the rotation.

The present spin axis precession rate of Mars is close to resonance with some of the orbital precession frequencies [12]. That is the basic reason why the obliquity oscillations computed for Mars are so much larger than for Earth. Even quite small changes in the instantaneous spin precession rate can move Mars into and out of resonance, yielding significant deviations from the obliquity pattern that would be obtained for a static mass distribution.

Insolation changes determine mass fluxes, whereas torques depend on where the mass is, not how fast it is moving. As a result, mass distribution tends to lag behind the insolation pattern driving it. The torques associated with this phase lag can add a secular trend to the obliquity history [13,14,15,16].

Evolution of the spin pole direction is governed by

$$\frac{ds}{dt} = \alpha \left(\frac{a}{b} \right)^3 (n \cdot s) (s \times n)$$

where α is a scalar rate constant, proportional to J_2 , the oblateness of the mass distribution, and (a, b) are the semimajor and semiminor axes of the orbit.

The oblateness of the mass distribution is assumed to be completely determined by two processes: surface mass transport, due to climate change, and internal flow of mantle material, in response to surface loads. The rate of internal flow will depend on the amplitude of the load, and on the density, rigidity and viscosity structure within Mars. For simplicity, assume a homogeneous body with Maxwell viscoelastic rheology. In that case variations in the precession rate parameters are given by

$$\frac{d\mathbf{a}}{dt} = (1+k) \frac{d\mathbf{a}_c}{dt} - \left(\frac{\mathbf{a} - \mathbf{a}_h}{\tau_v} \right)$$

where k is an elastic load Love number, which depends on density and rigidity, τ_v is a viscous relaxation time, and (α_c , α_h) are precession rate values corresponding to the ice load (cap and/or ring) and hydrostatic equilibrium, respectively. If the relaxation time τ_v is short, compared to obliquity variations, the value of α will remain close to hydrostatic. If the viscosity is Earth-like (10^{21} - 10^{22} Pa s), the viscous relaxation time will be (6-60) 10^3 years.

The complete model has six free parameters. The climatic response is determined by four parameters (m_i , ϵ_1 , ϵ_2 , τ_c) and the mantle flow response is given by two parameters (k and τ_v). None of these parameters are well known, though the mantle flow parameters

are arguably close to those obtained for Earth-like rigidity and viscosity. The secular obliquity drift rate depends sensitively on the phase lag of precession rate versus obliquity. By varying the parameters over their plausible ranges it is easy to generate a suite of obliquity histories and ice cap deposition histories. Even modest changes in some of the input parameters lead to rather significant variations in the model output.

It has been claimed that the obliquity of Mars varies chaotically [5,17]. That claim is based on calculations with a fixed spin axis precession rate parameter. The model presented here introduces dissipation into the system.. A frequent consequence of adding dissipation to an otherwise chaotic system is the emergence of a strange attractor. The nature of the Mars obliquity attractor is not yet known.

References: [1] Ward, W.R. (1974) *JGR*, 79, 3375-3381. [2] Ward, W.R. (1979) *JGR*, 84, 237-241. [3] Bills, B.G. (1990), *JGR*, 95, 14,137-14,153. [4] Hilton, J.L., (1992) *Astron. J.* 103, 619-637. [5] Touma, J. and J. Wisdom, (1993) *Science* 259, 1294-1297. [6] Pollack, J.B. (1979) *Icarus*, 37, 479-553. [7] Fanale, F.P. et al (1982) *Icarus* 50, 381-407. [8] Francois, L.M. et al. (1990) *JGR* 95, 14,761-14,778. [9] Clifford, S.M. et al. (1988) *Eos*, 69, 1585-1596. [10] Laskar, J. (1988) *Astron. Astrophys.* 198, 341-362. [11] Jakosky, B.M. and M.H. Carr (1985) *Nature* 315, 559-561. [12] Ward, W.R., and D. Rudy (1991) *Icarus* 94, 160-164. [13] Rubincam, D.R. (1990) *Science* 248, 720-721. [14] Rubincam, D.R. (1993) *JGR* 98 10,827-19,832. [15] Bills, B.G. (1994) *GRL*, 21, 177-180. [16] Spada, G., and L. Alfonsi, (1998) *JGR*, 103, 28,599-28,605. [17] Laskar, J., (1993) *Nature*, 361, 608-610.