KINETICS OF INELASTIC SCATTER OF ELECTROMAGNETIC WAVES BY LANGMUIR WAVES IN AN INHOMOGENEOUS COLLISIONLESS PLASMA

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Abstract. A development is initiated of informative version of decay of electromagnetic wave on another one and the Langmuir wave. The kinetics of drift of electromagnetic waves in space and wavenumbers due to the plasma inhomogeneity for this new understanding of the phenomenon is reported.

1. Introduction

For characterizing the degree of adequateness of a plasma theoretical scenario to the real picture of the plasma physical evolution, a term *informativeness* was introduced [1–7]. Conceptually, the longer the scenario depicts faithfully the objective picture of the plasma macrophysical evolution, the higher an estimate the researcher should suggest for the scenario informativeness. The approaches of traditional plasma theory cannot help one to develop appropriately informative plasma physical scenarios: Common methods of the theory yield equally rigorous justifications to incompatible versions of the same physical phenomenon [1,2, 4–6,8–12]. There are two basic inseparable reasons of *non-informativeness* of the theory: An absence of proper understanding of significance of asymptotic nature of convergence of successive approximations to the plasma scenario, and The tradition of substitution of real plasmas by probabilistic plasma ensembles (the ensemble method). We comment that the choice of the leading order approximation of asymptotically converging iterations defines the conditional limit of the theory and hence comprises corresponding version of the plasma physical scenario. Equally, the interplay of the plasma ensemble statistics strongly depends on the ensemble content, wherefore its analysis does not help to uncover the objective picture of the plasma macrophysical evolution. Finally, we have explained how the top informativeness of the scenario depends on the theory expansion parameter [1,2,7].

Above clarified disposition is quite natural from the viewpoint of easy provisions of the information theory. First, data on positions and momenta of individual plasma particles is always incomplete. Second, a full-scale account for such a data would not have been technically possible, because it is equivalent to integration of immense numbers of motion equations of individual particles. Therefore, theorists do inevitably appeal to simplified plasma models that can follow the real plasma macrophysics during a finite time intervals. Hence, the problem arises of developing models that possess by possibly longer periods of correspondence to real plasma behaviors. This motif should be laid in the basement of the theory development.

The basic branch of the plasma theory is the plasma kinetic theory: it underlies all the others. Respectively, the problem of gaining the theory turns to a problem of developing high-informative plasma kinetic models. One can form the latter from full plasma description via its careful reducing[1]. We speak about care because one should accurately select the information basis of his theory from the full plasma data and dismiss the rest of the latter. Will be his (her) choice successful, it will result in developing more informative plasma physical scenario. Above unrolled reasons of theory non-informativeness suggest basic principles for reaching the most informative of possible plasma kinetic models [1,2,6,12]. First, the researcher should refrain from the plasma ensemble substitution. This forces to modify the concept of the distribution function of plasma particles. Mathematically, the distribution function represents some statistic of distribution of discrete charged particles in $\vec{r} - \vec{p}$ phase space. Usual approaches implied the developing of such a statistic, the distribution function of Vlasovian type $f_\alpha(\vec{r}, \vec{p}, t)$ [13], just via the ensemble averaging of its counterpart from the full plasma description, $N_\alpha = \sum_n \delta^3(\vec{r} - \vec{r}_n(t))\, \delta^3(\vec{p} - \vec{p}_n(t))$ [14,15][2]. The only possibility of evading from such an averaging consists in its substitution by a contextually oriented averaging in phase space of positions and momentums of plasma particles[3].

[1] The full description of a classical ionized plasma is given by simultaneous Maxwell and Klimontovich– Dupree equations. They contain cumulatively the motion equations of all individual charged plasma particles.

[2] The subscript n numbers particles of the given species α, either electrons ($\alpha \to$ e) or ions ($\alpha \to$ i), and functions $\vec{r}_n(t)$ and $\vec{p}_n(t)$describe trajectories of individual plasma particles

[3] An appropriate arrangement of the averaging depends essentially on the physical problem under consideration

Second principle of developing informative plasma kinetic scenarios is that the researcher should develop its successive iterations using a direct time integration of necessary evolutional equations. On this path he can properly account for an available information on current plasma state and its recent history and simultaneously discriminate an indeterminate information on time remote plasma states [2,6, 12].

The practice of plasma studies consists of many activities whereat the heightening of informativeness of plasma scenarios is strongly desirable. We have dwelled upon a particular one of nonlinear phenomena in turbulent plasmas[4] and created for their modeling the *high-informative correlation analysis of plasma kinetics* [12]. We suggested its generalization onto a case of turbulent plasmas with fields of solenoidal waves, apart from the former potential ones only, in Ref. 7. We developed there the high-informative kinetic scenario of nonlinear transformation of Langmuir waves to electromagnetic ones. We have shown that the usal understanding of this phenomenon [16–18] distorts substantially the picture of merging the long Langmuir waves. This fact originates from the fundamentals of traditional nonlinear plasma theory. Basically, our predecessors relied upon the random phase approximation[5]. It was deduced to substantiate "the golden rule"[6], which permitted to calculate the wave collision integral in some easy context of wave interactions and then generalize the result onto a common case.

In the current paper, we report generalization of our approach onto *inhomogeneous* plasmas with turbulent fields of solenoidal waves. We shall develop on its basis to the opening of MSS-14 a scenario of the decay of high-frequency electromagnetic wave on another electromagnetic wave and the Langmuir one. This process otherwise can be called *an inelastic scatter of electromagnetic waves by Langmuir waves*. To the current moment, we uncovered respective picture of the

[4] Just the controversies of nonlinear plasma theory has helped us to highlight the problem of theory informativeness that is intrinsic not for the plasma physics only

[5] Conceptually, the random phase approximation should be qualified as a particular realization of the plasma ensemble substitution

[6] The rule supposed that the numbers of interacting waves entered the wave collision integral in some symmetric combination. It was developed on a basis of either the semiheuristic reasonings with appeal to a limit of thermodynamic equilibrium of a medium with an electromagnetic radiation [27,28, pgs. 411–413] or the calculations of semiquantum character that explored means of secondary quantization from the quantum field theory [29, 30].

turbulence drift in space and wavevectors due to the plasma inhomogeneity. In view of lack in space, we can not unroll the details of respective theoretical motion. We restrict ourselves with comparison of anew developed picture of drift of electromagnetic waves with its traditional analog.

2. Basic features of natural oscillations in turbulent plasmas

The wave field in a turbulent plasma is characterized by a two-time correlation function $\Phi^{ijkl}(\vec{r},t,\vec{r}',t')$, the product of electromagnetic field tensors F^{ij} at differing times at spatial points with fixed displacement that is averaged over the surfaces of constant plasma macroscopic density, $\Phi^{ijkl}(\vec{r},t,\vec{r}',t') = \langle F^{ij}(\vec{r},t)F^{kl}(\vec{r}',t')\rangle_{\vec{r}-\vec{r}'=\mathrm{const}}$. It possesses by a uniquely defined spatial Fourier transform,

$$\boldsymbol{\Phi}_{\vec{k}}(\vec{r},t,t') = \int \frac{d^3\vec{R}}{(2\pi)^3}\boldsymbol{\Phi}\left(\vec{r}+\frac{\vec{R}}{2},t,\vec{r}-\frac{\vec{R}}{2},t'\right)\exp(-\mathrm{i}\vec{k}\vec{R}).$$

The bulk of the latter comprises the plasma natural oscillations,

$$\Phi_{\vec{k}}^{ijkl}(\vec{r},t,t') = \sum_{\sigma,s}\Phi_{\vec{k}}^{\sigma s\,ijkl}(\vec{r},t,t')\,;$$

the most natural choice of the leading order of $\Phi_{\vec{k}}^{\sigma s\,ijkl}(\vec{r},t,t')$ for the context of three-wave interactions is given by

$$\begin{aligned}&{}^{0}\Phi_{\vec{k}}^{\sigma s\,ijkl}(\vec{r},t,t')\\ &= F_{\vec{k}}^{\sigma s\,ij}(\vec{r})\left(F_{\vec{k}}^{\sigma s\,kl}(\vec{r})\right)^{*} n_{s\vec{k}}^{\sigma}(\vec{r},t')\exp\left(-\mathrm{i}\int\omega_{\vec{r}}^{\sigma s}(\vec{r},\tau)d\tau\right),\\ &t>t',\\ &{}^{0}\Phi_{\vec{k}}^{\sigma s\,ijkl}(\vec{r},t,t') = \left({}^{0}\Phi_{\vec{k}}^{\sigma skl ij}(\vec{r},t',t)\right)^{*}, \qquad t<t'.\end{aligned}$$

Here $\omega_{\vec{r}}^{\sigma s}(\vec{r},t)$ is a renormalized wave natural frequency that consist of real wave frequency and imaginary nonlinear wave damping rate, $\omega_{\vec{r}}^{\sigma s}(\vec{r},t) = s\omega_{s\vec{k}}^{\sigma}(\vec{r},t) - i\gamma_{s\vec{k}}^{\sigma}(\vec{r},t)$. Tensor $\boldsymbol{F}_{\vec{k}}^{\sigma s}$ is the wave polarization tensor, σ denotes the wave polarization, and $s=\pm$ distinguishes the wave and its counterpropagating twin. Feature $n_{\vec{k}}^{\sigma}$ stands for real non-negative *wave spectral density*.

Above idea, of the leading order of two-time correlation function, was developed for homogeneous plasmas. Generally, it should be modified for inhomogeneous plasmas. Fortunately, modification is not necessary for electromagnetic waves in a plasma with small thermal and magnetic wave dispersions[7].

3. Formula for the wave drift in phase space

For ease of comparing of our findings with a traditional knowledge, we unroll correspondence of our wave spectral density with former wave number density $N_{\vec{k}}^{\sigma}$,

$$N_{\vec{k}}^{\sigma} = \frac{2\pi^2}{\hbar\omega_{\vec{k}}^{\sigma}} \frac{1}{\omega} \frac{\partial}{\partial\omega} \left(\omega^2 \left(\varepsilon_{\vec{k}\omega}^{\beta\gamma}\right)^{H}\right)_{\omega=\omega_{\vec{k}}^{\sigma}} \left[n_{\vec{k}}^{\sigma} \left(F_{\vec{k}}^{\sigma+}{}_{0\beta}\right)^{*} F_{\vec{k}}^{\sigma+}{}_{0\gamma}\right].$$

In neglect by thermal and magnetic wave dispersions, this yields

$$N_{\vec{k}}^{EM\pm} = \frac{4\pi^2}{\hbar} \frac{\sqrt{\omega_{pe}^2 + k^2c^2}}{\omega_{pe}^2 + 2k^2c^2} n_{\vec{k}}^{EM\pm}.$$

In terms of $N_{\vec{k}}^{EM\pm}$, our picture of wave drift in space and wavevectors is governed by

$$\frac{\partial}{\partial t} N_{\vec{k}}^{EM\pm} = \left(\frac{\partial\omega_{\vec{k}}^{EM\pm}}{\partial\vec{r}} \frac{\partial N_{\vec{k}}^{EM\pm}}{\partial\vec{k}}\right) - \left(\frac{\partial\omega_{\vec{k}}^{EM\pm}}{\partial\vec{k}} \frac{\partial N_{\vec{k}}^{EM\pm}}{\partial\vec{r}}\right) + N_{\vec{k}}^{EM\pm} \frac{\omega_{pe}^2}{2\omega_{\vec{k}}^{EM\pm}(\omega_{pe}^2 + 2k^2c^2)} \left(\frac{\partial\omega_{\vec{k}}^{EM\pm}}{\partial\vec{k}} \frac{\partial\omega_{\vec{k}}^{EM\pm}}{\partial\vec{r}}\right).$$

The bottom line here constitutes the difference of our image of the process with its traditional version (that is known from the geometrical optics). We assume that its appear results from more profound account of the correspondence of electric and magnetic fields in the wave polarization tensor at their differentiations in $\vec{r}$ and $\vec{k}$.

References

1. V.I. Erofeev.// J. Plasma and Fusion Research Series, 2009, v.8, p.59.
2. V.I. Erofeev.// Transactions of Fusion Science and Technology, 2011, v.59, p. 316.
3. V. Erofeev. *Printsipy razrabotki visokoinformativnikh modelei plazmennoi kinetiki* (Publishing House of the Siberian Branch of

[7] The case is different for Langmuir waves in the same plasma

RAS, 2013), ISBN 978-5-7692-1307-6, URL http://www.sibran.ru/catalog/FM/150268.
4. V.I. Erofeev.// Physics of Plasmas, 2004, v.11, p.3284.
5. V.I. Erofeev.// J. Fusion Energy, 2010, v.29, p.337.
6. V.I. Erofeev.// J. Fusion Energy, 2011, v.30, p.157.
7. V.I. Erofeev.// J Plasma Phys., 2014, v.80, p.289.
8. V.I. Erofeev.// J. Fusion Energy, 2000, v.19, p.99.
9. V.I. Erofeev.// Phys. Plasmas, 2002, v.9, p.1137.
10. V.I. Erofeev.// J. Fusion Energy, 2003, v.22, p.259.
11. V.I. Erofeev.// J. Plasma Phys., 2004, v.70, p.251.
12. V. Erofeev. *High-informative plasma theory* (LAP, Saarbrücken, 2011).
13. A.A. Vlasov.// J. Phys. (U.S.S.R.), 1945, v.9, p.25.
14. Y.L. Klimontovich.// Sov. Phys. JETP, 1958, v.6, p.753.
15. Y.L. Klimontovich. *The Statistical Theory of Nonequilibrium Processes in a Plasma* (MIT Press, Cambridge (Mass.), 1967).
16. Y. Terashima & N. Yajima.// Progr. Theor. Phys., 1963, v.30, p.443.
17. L.M. Al'tshul' and V.I. Karpman.// Sov. Phys. JETP, 1965, v.20, p.1043.
18. I.A. Akhiezer, I.A. Danelia, and N.L. Tsintsadze.// Sov. Phys. JETP, 1964, v.19, p.208.
19. A. Akhiezer, I. Prokhoda, and A. Sitenko.// Sov.Phys. JETP, 1958, v.6, p.576.
20. M. Rosenbluth and N. Rostoker.// Phys. Fluids, 1962, v.5, p.776.
21. V.N. Tsytovich. *Nonlinear Effects in Plasma* (Plenum, New York, 1970).
22. L. Kovrizhnykh and V. Tsytovich.// Sov. Phys. Doklady, 1965, v.9, p.913.
23. V.I. Erofeev.// J. Plasma Phys., 1997, v.57, p.273.
24. V.N. Tsytovich.// Sov. Phys. Uspekhi, 1967, v.9, p.805.
25. T.H. Dupree.// Phys. Fluids, 1963, v.6, p.1714.
26. R.Z. Sagdeev and A.A. Galeev. *Nonlinear Plasma Theory* (Benjamin, New York, 1969).
27. V.L. Ginzburg and V.V. Zhelezniakov.// Sov. Astron. J., 1958, v.2, p.653.
28. V.V. Zhelezniakov. *Elektromagnitnye volny v kosmlcheskoi plazme* (Nauka, Moscow, 1977).
29. A.A. Galeev and V.N. Karpman.// Sov. Phys. JETP, 1963, v.17, p.403.
30. L.M. Kovrizhnykh.// Sov. Phys. JETP, 1965, v.21, p.744.